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A manufacturer-service provider model with remanufacturability and variable product life considerations

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and variable product life considerations**

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Abstract

There has been a growing emphasis on remanufacturing as a profitable means to reduce wastage, conserve energy and costs. An alternate approach to obtain similar environmental and economic benefits is to increase the product life. We model and analyze the economic relationships among the level of remanufacturing, product life and economic consequences under the framework of a manufacturer/remanufacturer and a service provider who utilizes the manufacturer's product to provide service to his/her customers. In our framework, the remanufacturability is defined as the fraction of used products that can be economically remanufactured, and it is assumed that the remanufacturability can be increased via fixed cost investment in product and process design technologies. We also assume that the product life which is defined to be the number of units of service that is provided from the product can be increased by utilizing higher quality components with corresponding higher variable cost. Under these assumptions, we formulate three distinct supply chain scenarios. Namely, a manufacturer driven supply chain, a centrally coordinated supply chain and a service provider driven supply chain. From the subsequent equilibrium and optimality analysis, we derive several interesting managerial insights. For example, there are several conditions under which a higher technology investment in remanufacturability leads to a shorter product life.

1. Introduction

1.1 Research Objectives and Overview

There has been a growing emphasis on remanufacturing as a profitable means to reduce waste conserve energy and save costs. Lund [1] defines remanufacturing as an industrial process in which worn-out products are restored to a like-new condition. In 2004, Xerox a copy machine manufacturer claimed to have saved 320 000 megawatts of energy and diverted more than 120 million pounds of material from the land fill via remanufacturing [2]. A key characteristic of remanufacturing environments is remanufacturability defined as the fraction of returned products that can be economically remanufactured [3]. The remanufacturability may be improved via investment in product and process design [3, 4].

An alternative approach to obtain similar environmental and economic benefits is to increase the product life. By product life, we mean the number of units of service provided by the product. The environmental protection agency recommends both a longer product life and product reuse to reduce wastage [5]. The product life may be extended by utilizing higher quality components that result in a greater variable cost [6]. The environmental protection agency recommends both a longer product life and product reuse to reduce wastage [5].

The purpose of this study is to model and analyze the economic relationships among the level of remanufacturing product life and economic consequences under the framework of a manufacturer/remanufacturer and a service provider who utilizes the manufacturer's product to provide service to her customers.

The specific research objectives are,

1) To derive the relationships between the remanufacturability and product life in the context of a manufacturer-service provider supply chain.

2) To investigate how supply chain coordination between the manufacturer and service provider impacts the remanufacturability, product life, profits and prices.

3) To analyze the impact of an environmental legislation that penalizes disposal of products with respect to the level of remanufacturing and product life.

Supply chains with both forward and reverse flows of product are referred to as closed loop supply chains. Savaskan [7] compared various closed loop supply chains structures where goods are sold to the customers and the product returns are driven by collection efforts. Another closed loop supply chain that is gaining relevance in the recent years is where services are sold to customers instead of goods.

Several remanufacturable products are being utilized to provide service to the customers. For example, copy machines are utilized by document centers such as Kinko's and Staples. Automotive transmission flush machines, commercial truck tires, washing machines and fitness equipment are all examples of remanufactured products being utilized to provide service to the customers. Electrolux a washing machine manufacturer/remanufacturer conducted a pilot project in Sweden to offer laundry service on a contract with about 1 dollar per laundry for 1000 washes as an alternative to selling the equipment [8].

Several articles suggest that the service selling supply chain where services, instead of goods, are sold to the customers promotes both product life as well as the level of remanufacturing [6, 9]. Stahel [10] argues that, selling of services places the emphasis on meeting the demand

for those services in the most cost effective manner and hence leads to a greater product life and product reuse compared to the traditional selling of goods. A key difference between service selling supply chain and the traditional supply chain is the collection efforts required for the return of used products. In case of service selling, manufacturers often lease the products to service providers who return without cost when the lease expires (e.g, Xerox copy machines and Pitney Bowes mail processing machines). In contrast, collection efforts such as advertisements, collection centers, and return fees may be required to get back products sold to individual customers.

We investigate the impact of supply chain coordination on the remanufacturability, prices and product life under a game theoretic framework. For this study, we focused on the decentralized model where the manufacturer is assumed to possess the supply chain power over the service provider as our basic model. The assumption of a dominant manufacturer is frequently observed in the supply chain literature and is based on belief that downstream supply chain members such as retailers and service providers are often smaller in size and operate in specific local markets [11, 12]. We formulated this supply chain as a Stackelberg game with the manufacturer as the leader. The Stackelberg game is appropriate for modeling a dominant supply chain member as it typically results in higher profits to the leader due to the advantage of choosing his/her strategies first [11-14].

1.2 Enhancing Remanufacturability via Technology Investment

An important feature of our model is the option for the manufacturer to invest in remanufacturability, the fraction of products that may be economically remanufactured (in this paper, we will use the terms remanufacturability and the level of remanufacturability (R)

interchangeably). By an economically remanufactured product, we mean that a specified amount of cost saving is achieved over the cost of manufacturing. The justification for such an investment in remanufacturing is three fold.

1) Design for remanufacturing: This involves improving the product design so as to facilitate the various steps involved in the remanufacturing process such as disassembly, product testing, cleaning, and reassembly. A product that is modular in design, easy to disassemble and reassemble may have a greater chance of being economically remanufactured as compared to product which is too expensive to disassemble. The various aspects of design for remanufacturing are discussed in Amezquita *et al* [3].

2) Improved remanufacturing process: Investment in remanufacturing equipment and processes may facilitate the remanufacturing of product subassemblies that are too difficult or expensive to remanufacture. Sundin [15] studied remanufacturing process of several companies and found that cleaning and inspection were often the most time consuming steps of the remanufacturing process. Automation of these steps was recommended to reduce the processing time. Recently Xerox replaced its traditional cleaning technology with carbon dioxide blasting which lead to reduced cleaning times as well as improved part recovery rates [16].

3) Improved Used Product Testing: Improved product testing enables the cost efficient identification of the remaining life of a product and will thus help in remanufacturing the product economically. For instance, Xerox has invested in the development of an advanced testing technology called signature analysis that facilitates the assessment of the remaining life of the used products as well as to remanufacture efficiently. The technology compares

the noise, heat and vibration levels of the used parts with the characteristics of the new parts to assess the condition of the used parts. The company claims that the technology has saved used parts with remaining life from being discarded based on the average performance[2].

Debo *et al* [10] consider similar fixed cost investment in remanufacturing. In what follows, we will discuss how the product life can be varied by utilizing better quality components resulting in higher variable cost.

1.3 Variable Product Life

Products in general are composed of multiple components that vary with respect to their component life. By component life, we mean the number of units of service that a component can provide before being worn out. For instance, moving parts in a copy machine such as bushings tend to wear out faster per unit service compared to the electronic components such as memory chips [6]. The product life ends when the shortest life component wears out. Under the circumstances, the product life can be increased by replacing relatively shortest life components with those having a higher component life. Fuji Xerox [17] utilized bushings with better thermal and frictional properties to extend product life. We consider that replacing the short life components with longer life components results in a higher variable cost [18, 19]. While we focus on the case where product life is increased by utilizing longer life components, the product life may also be improved technology investments in a more robust design.

1.4   Environmental Fee Based on a Type of E-Waste Legislation

As part of this study we also examined how a specific type of environmental legislation that is aimed at reducing the waste impacts the remanufacturability and product life. Specifically, we consider the scenario where the government imposes an environmental fee for each product that is disposed into the waste stream. The fee σ can be utilized for various purposes such as recycling, environmental disposal and consumer education. This fee structure is influenced by the recent e-waste legislation in Maine where the producers pay for recycling of their products in details [20].

1.5   Service Provider Driven Supply Chain

In recent years, the service provider driven supply chain (SPDSC) has gained relevance as service providers are becoming increasingly powerful by organization of multiple service centers into service provider chains. For example, large service provider chains such as Fedex Kinkos and Staples are likely to have a considerably high supply chain power. Although retailers and service providers are becoming more dominant relative to the manufacturer, only a small minority of papers in the supply chain literature have considered a dominant downstream supply chain member [12, 13, 21]. We formulate SPDSC as a Stakcelberg game based upon service provider's margin as a benchmark and compare with the equilibrium solution of MDSC.

Our results show that there is a tradeoff between product life and remanufacturability and that parameters that are leading to a higher remanufacturability are also resulting in a lower product life. We also found that the decentralized manufacturer driven supply chain has a lower remanufacturability and a longer product life compared to the centrally coordinated

supply chain. When product life is constant, we found that an environmental fee penalizing product disposal can lead to decreased remanufacturing and a higher price depending on the parameters. Finally, given constant product life we found the service provider driven supply chain to be superior to the manufacturer driven supply chain with respect to remanufacturability and price.

Given this overview, the rest of the paper is organized as follows. In Chapter 2, we review the relevant literature in vertical supply chains, remanufacturing and product life extension. Next in Chapter 3, we formulate the manufacturer driven supply chain MDSC with investments in remanufacturability and constant product life as a Stackelberg game and determine the equilibrium conditions. In addition, we compare the MDSC equilibrium with the centrally coordinated supply chain CCSC optimal solution as benchmark. Next, in Chapter 4, we extend MDSC to consider investment in remanufacturability as well as a variable product life and compare with the corresponding CCSC optimal solution. Next in Chapter 5, we analyze how an environmental fee imposed on each disposed product impacts the MDSC equilibrium. In Chapter 6, we consider the case when the service provider possesses the supply chain power as a Stackelberg game and compare with CCSC and MDSC. Finally, we provide concluding remarks and directions for further study in Chapter 8.

2. Literature Review

This research is associated with three streams of literature which address remanufacturing, variable product life and vertical supply chain. In what follows, we first discuss the remanufacturing literature followed by the product life and vertical supply chain literature.

Traditionally, the remanufacturing literature focused on tactical and operational issues such as optimizing the inventory levels and disassembly sequences of returned products[22, 23]. Recently, there has been a shift in emphasis towards the strategic issues such as the level of technology investments, pricing of new and remanufactured products, competition with independent remanufacturers etc. Production cost savings over the cost of manufacturing a new product is a key economic drivers of remanufacturing [24]. Given that remanufacturing requires fixed cost technology investments, the profitability of remanufacturing can depend on the volume of used products over which the production cost savings can be realized. Debo *et al* [4] investigated the economic drivers behind remanufacturing when the remanufactured product is perceived to be of a lesser quality then the new product. Similar to our model, they consider that remanufacturability can be improved via investments in process and product design technologies. They found that the profitability of remanufacturing depended on the production cost savings, the market size and the market heterogeneity.

Given that closed loop supply chains have both forward and reverse flows, the supply chain interactions with intermediaries such as retailers, and third-party collectors can have a significant impact on the degree of remanufacturing. Savaskan *et al* [7] analyze the impact of supply chain configurations and gaming behavior between the members on the investment in

collection effort when products are sold to the customers. Utilizing a linear demand and quadratic investment function for collection effort they discuss how the lack of supply chain coordination can result in lower investment in remanufacturability. We similarly consider that fixed cost investments can improve the fraction of products that are remanufactured. However, we focus on a service selling framework, variable product life and more general investment functions.

We will now review the relevant articles on a variable product life. Several articles analyzed the impact of durability from an economic perspective. In these models, the durability is often modeled as a function of the variable cost of manufacturing. Levhari and Srinivasan [18] compare the durability under monopoly and perfect competition when the durable goods are utilized to provide service. They formulate a discounted profit model with constant production quantity in each period and conclude that durability would be lesser under monopoly as compared to competition. Swan[16] contends that the Levhari and Srinivasan [17]'s model has a specification error and that the choice of durability is based on minimizing the cost of providing a given flow of services and is independent of the demand and revenue conditions. Swan concludes that the monopolist will have the same durability as under competition while producing a lower quantity of goods and charging higher per unit service. Similar to the Swan [16] model, the optimal product life given the remanufacturability is independent of the demand parameters except for their indirect effect via the remanufacturability in our model.

A central concept in the pricing models of vertical supply chain is the phenomenon of double marginalization. Double marginalization refers to the loss of profits and higher retail price in

a decentralized supply chain because of two successive mark-ups. Double marginalization occurs because the retailer does not take manufacturer's profit into account while setting his retail price [25]. The double marginalization in vertical supply chains is similar to the classical prisoner's dilemma in game theory and may result in decreased quality of goods in addition to the increased prices [26]. Considering the double marginalization, distributing via a company owned store is more profitable than utilizing supply chain intermediaries such as retailers when the supply chain is a monopoly (often referred to as bilateral monopoly in the supply chain literature). When competition is considered, as shown by Mcguire and Staelin [11], utilizing supply chain intermediaries can be more profitable compared to a company owned store as the intermediaries act as a buffer to reduce the extent of competition between two manufacturers. However, the price to the customers is higher than in a coordinated supply chain in competition as well.

The decreased demand in decentralized supply chains due to double marginalization can result in reduced incentive for production cost saving technology investment. Gupta and Loulou [27] further extend Mcguire and Staelin [11] model to consider production cost saving technology investment. They assume a linear demand function and a quadratic investment function relating the production cost savings to the technology investment. Under the circumstances, they show that double marginalization in an uncoordinated supply chain results in lower technology investment regardless of the product substitutability. In our model, a specified production cost savings is achieved over the manufacturing cost per every remanufactured product. The manufacturer driven supply chain (MDSC) in our model is closely related Gupta and Loulou's model in that technology investment can lead to reduced

average production cost per product in both the models.   We extend Gupta and Loulou's model in the following aspects: 1) our results are applicable for a more general class of investment functions. 2) we consider the case when product life is variable in addition to the fixed cost investments in remanufacturability 3) we analyze the impact of an environmental fee imposed on each disposed product 4) We formulate the service provider driven supply chain (SPDSC) as an important benchmark and perform comparative analysis.

3. The Manufacturer Driven Supply Chain (MDSC)

In this chapter, we formulate and analyze a decentralized model of the manufacturer service provider supply chain assuming that the manufacturer possesses a greater supply chain power over the service provider. The assumption of a dominant manufacturer is frequently observed in the supply chain literature and is based on belief that downstream supply chain members such as retailers and service providers are often smaller in size and operate in specific local markets [11, 12]. Considering this assumption, MDSC is more relevant for the case of small and independent service providers. We formulate this supply chain as a Stackelberg game with the manufacturer as the leader. The Stackelberg game is appropriate for modeling a dominant supply chain member as it typically results in a monetary advantage to the leader due to choosing his/her strategies first [13, 16, 28].

We perform comparative statics to analyze how various parameters impact the equilibrium remanufacturability and the price in the above Stackelberg game. Such analysis is useful in identifying conditions under which a greater investment in remanufacturability is justified. We also compare MDSC equilibrium with centrally coordinated supply chain (CCSC) as a benchmark to analyze how the double marginalization in MDSC impacts the optimal prices and remanufacturability.

The chapter is organized as follows. In Section 3.1, we define notations, provide an overview of MDSC and discuss our assumptions. Next in Section 3.2, we formulate MDSC as a Stackelberg game and derive the equilibrium conditions. Next in Section 3.3, we analyze the sensitivity of the equilibrium with respect to key parameters. Next in Section 3.4, we compare MDSC equilibrium with CCSC solution as a benchmark. Finally in Section 3.5, we

utilize a specific quadratic investment function to illustrate the analytical results and make further observations. The notations utilized in this paper are summarized in Appendix A. The results for the forward supply chain where remanufacturing is not modeled are provided in Appendix B for comparison.

3.1. Description of the Supply chain and Assumptions

The MDSC is shown in Figure 1. The MDSC consists of a manufacturer who manufactures as well as remanufactures his products and a service provider who in turn utilizes the manufacturer's products to provide service to her customers. The service provider charges a price *p* per unit service. By a unit of service, we mean a quantifiable measure of service for which the service provider charges a fixed price. For example, a single copy or a load of clothes in a laundry are both units of service. Given *p,* the service provider faces a demand of $D(p)=\beta-\gamma p$, $0<p<\beta/\gamma$ units of service where $\beta>0$ is the maximum demand for services and $\gamma>0$ is the marginal demand and denotes the decrease in demand for a unit increase in price *p* (step 1 in Figure1). Linear demand functions have been widely utilized in supply chain literature [7, 11, 21, 29]. While the linear demand assumption enables us to identify and characterize the key supply chain phenomena, further study is required to generalize the results to a wider class of functions.

We consider that each product provides *α, α > 0* units of service from its manufacture / remanufacture to when it is collected as a used product for possible remanufacturing. The parameter *α* (units of service/product) represents the product life e.g., the number of copies provided by a copier in its life. Hence, to meet the demand *D(p),* the service provider

requires $D(p)/\alpha$ products which she purchases from the manufacturer at a wholesale w (step 2 in Figure 1).

We assume that the service provider returns all of the $D(p)/\alpha$ products at the end of use at no cost to the manufacturer (Step 3 in Figure 1). The justification for this assumption is twofold: 1) leased products such as copy machines (manufactured by Xerox), mail processing equipment (manufactured by Pitney – Bowes) are often returned to the manufacturer without an additional cost at the end of lease; 2) mathematically, two separate prices for sale of new products and return of used products can equivalently be represented by a single price as long as all the products are returned regardless of price. To elaborate, suppose that the forward wholesale price per product is w_1 and the manufacturer paid a collection fee A per every returned product. The service provider's profit would then be $D(p)(p - w_1/\alpha + A/\alpha)$. In this case, combining the two prices w_1 and A into a single wholesale price w will result in an equivalent profit maximization problem for the service provider.

The manufacturing cost per product is C_m dollars per product while the remanufacturing cost per product is C_r. Given that production cost savings is the primary economic motive for remanufacturing in the U.S.A [1], we assume that the remanufacturing cost C_r is less than the manufacturing cost C_m per product by a fixed amount Δ, $\Delta < C_m$. In practice, it is uneconomical to remanufacture all the returned products as they vary in their condition. We define remanufacturability R, $0 < R < 1$ as the fraction of returned products that can be economically remanufactured to realize the cost savings Δ. Under the circumstances, the manufacturer remanufactures a fraction R of the $D(p)/\alpha$ returned products (Step 4 of Figure 1) and disposes the rest $(1-R) D(p)/\alpha$ products as being unsuitable for remanufacturing(Step 5

of Figure 1).
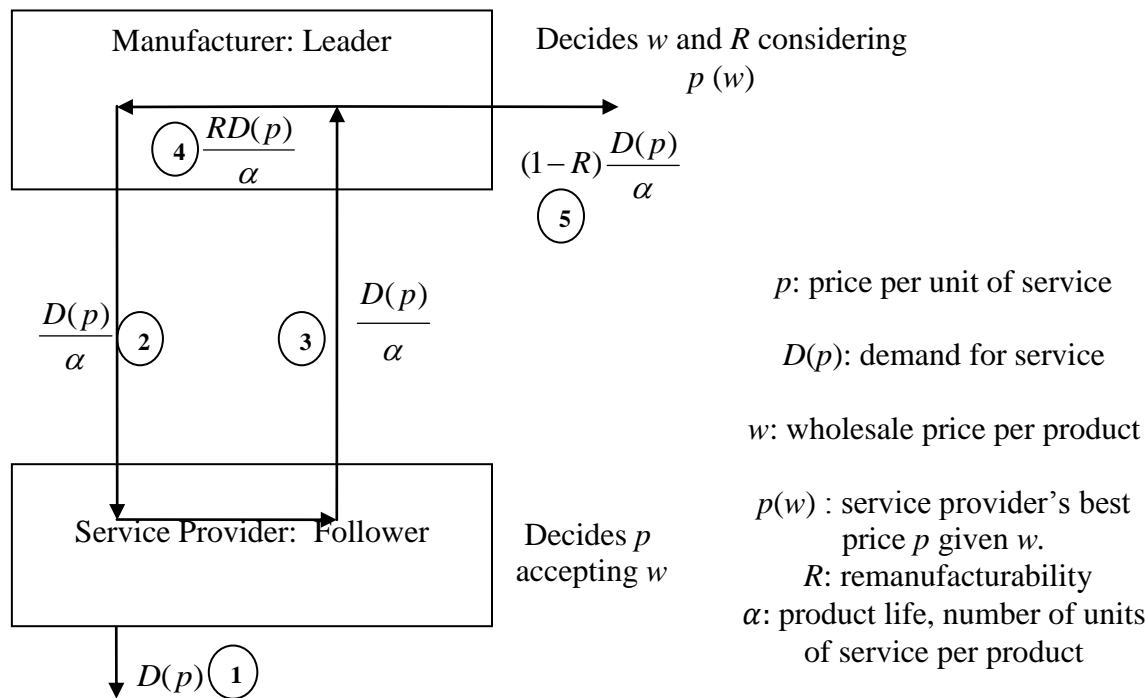
Figure 1 Manufacturer Driven Supply Chain

We consider that the manufacturer can increase the remanufacturability R by investing $I(R)$ (dollars) in product design, testing and remanufacturing process technologies. We assume that $I(R)$ is a convex and increasing function of R implying that increasing investments are required to obtain additional fixed increments in remanufacturability. Given this overview, we will now state the main assumptions of our model.

Assumption 1: The planning horizon is a single period representing the effective operation period of the remanufacturing technology corresponding to the investment $I(R)$. The price p,

the wholesale price *w,* and the remanufacturability *R* are all decided at the start of the single period and are held constant thereafter. We note that the cost savings from remanufacturing may be less initially until the products manufactured at the start of the period are returned. However, the initial portion's impact will be small if the single period's length is relatively large or if returned products from the previous period are available (i.e., a steady state perspective). We utilize the single period model as a basic step to identify the main economic implications of jointly optimizing remanufacturability and product life in a vertical supply chain and lay the groundwork for more detailed dynamic multi-period models [4, 19]. We note other articles in the literature made a similar assumption as well [27, 30].

Assumption 2*:* No difference between the quality of the manufactured and the remanufactured products. The validity of this assumption depends on the nature of the product. For instance, while the assumption may be reasonable for products such as copy machines that are remanufactured to extremely high standards, remanufactured automobile parts are often perceived to be of lesser quality.

Assumption 3: the parameters of the model are such that the optimal remanufacturability R^* satisfies $0 < R^* < 1$ and the optimal demands and the profits are greater than zero. We make this assumption to focus on the more relevant case of interior and feasible solutions. The assumption seems reasonable because in practice unusually high investments might be required if every returned product is to be remanufactured.

Assumption 4: the investment function $I(R)$ satisfies $I(R) > \gamma\Delta^2/2\alpha^2$. This assumption ensures that the total supply chain profits in CCSC and the manufacturer's profit in MDSC is concave with respect to R and p.

Assumption 5: the manufacturer and service provider constitute a bilateral monopoly, i.e., the manufacturer has a monopoly in the product and the service provider has a monopoly in the service offered. We make this assumption to analyze the impact of the basic supply chain interactions and the double marginalization on the remanufacturability. Validating the results under competition and alternate supply chain configurations is a relevant area for future research.

3.2 Stackelberg Game Formulation of MDSC

We model MDSC as a Stackelberg game where the manufacturer is the leader and makes his decisions first while the service provider is the follower and makes her decisions later. Supply chains with a dominant manufacturer are often modeled as a manufacturer leader Stackelberg game since it typically results in higher profits to the leader due to the advantage of moving first [13, 16, 28].

In a Stackelberg game with two players, player 1 first chooses his/her strategy x_1 (Stackelberg leader) and then player 2 observes this choice and chooses his/her strategy x_2. To determine the Stackelberg equilibrium $(x_1{}^* \quad x_2{}^*)$, we first determine player 2's strategy $x_2{}^*(x_1)$ that maximizes his/her profit for each value of player 1's strategy. Here $x_2{}^*(x_1)$ is called the best response function. We then determine the player 1, strategy $x_1{}^*$ that maximizes his profits $\pi(x_1, x_2{}^*(x_1))$ [31].

In MDSC, since the service provider is the follower, she accepts the wholesale price per product w that is determined by the manufacturer, and in turn determines the price per unit service $p(w)$ that maximizes her profits. Being the leader, the manufacturer anticipates the service provider's best response function $p(w)$ and determines the optimal wholesale price w_m and the remanufacturability R_m that maximize his profit. In what follows, we will derive sufficient conditions for the existence and uniqueness of the Stackelberg equilibrium and state the equilibrium conditions.

To solve the Stackelberg game we first optimize the service provider's profit π_{sp}^m and determine her best response function $p(w)$ to a given w. The service provider's profit maximization given the wholesale price w is formulated by (1). The term $(\beta - \gamma p)$ in (1) is the demand for services while $(p - \frac{w}{\alpha})$ is the service provider's margin per unit service.

$$\underset{p}{Max} \quad \pi_{sp}^m = (\beta - \gamma p)(p - \frac{w}{\alpha}) \tag{1}$$

In the Stackelberg game, concavity of the follower's objective implies that the best response function exists and is a sufficient condition for the existence of the Stackelberg equilibrium[31]. From (1), $\frac{\partial^2 \pi_{sp}^m}{\partial p^2} = -2\gamma < 0$. Therefore, π_{sp}^m is a concave function of the price p implying that the service provider's best response function is single valued. The first order condition for maximizing the service provider's profit is given by (2).

$$\frac{\partial \pi_{sp}^m}{\partial p} = \beta - 2\gamma p + \frac{w}{\alpha} \tag{2}$$

Solving (2) the service provider's best response function $p(w)$ is as provided by (3)

$$p(w) = \frac{\alpha\beta + w\gamma}{2\alpha\gamma} \tag{3}$$

The next step in solving the Stackelberg game is to determine the w and R that maximize the

manufacturers profit π_m^m while considering the service provider's best response

function $p(w)$. The manufacturer's profit maximization problem is formulated by (4). Here,

the term $(\beta - \gamma p(w))/\alpha$ is the total number of products while $(w - C_m + R\Delta)$ is the

manufacturer's margin per product. The fixed cost investment required to achieve a

remanufacturability of R is represented by $I(R)$.

$$\underset{w,R}{Max} \quad \pi_m^m = \frac{(\beta - \gamma p(w))}{\alpha}(w - C_m + R\Delta) - I(R) \tag{4}$$

Substituting the service provider's best response function $p(w)$ from (3) into (4),

$$\underset{w,R}{Max} \, \pi_m^m = \frac{(\alpha\beta - w\gamma)}{2\alpha^2}(w - C_m + R\Delta) - I(R) \tag{5}$$

A sufficient condition for the existence of the Stackelberg equilibrium is the concavity of the

leader's objective [31]. From (5), it can be verified that the manufacturer's profit function

π_m^m is concave in w and R if inequality (6) is true. We assume (6) is true to focus on the cases

where the Stackelberg equilibrium is unique.

$$\frac{\gamma\Delta^2}{4\alpha^2} < I''(R) \tag{6}$$

The equilibrium price w_m and R_m are found by equating the first derivatives of the

manufacturer's profit (7) and (8) to zero.

$$\frac{\partial \pi_m^m}{\partial w} = \frac{(\alpha\beta - w\gamma)}{2\alpha^2} - \frac{\gamma(w - C_m + R\Delta)}{2\alpha^2} \qquad (7)$$

$$\frac{\partial \pi_m^m}{\partial R} = \frac{(\alpha\beta - w\gamma)\Delta}{2\alpha^2} - I'(R) \qquad (8)$$

Considering the service provider's best response function (3), we can express the first order condition (8) as given by (9)

$$\frac{\Delta(\beta - \gamma p(w))}{\alpha} - I'(R) = 0 \qquad (9)$$

Therefore, from (9) the optimal remanufacturability only depends on the optimal demand for products $(\beta - \gamma p(w))/\alpha$ and the cost savings per remanufactured product Δ. This dependence of remanufacturability on the demand for products is because investment in remanufacturability is a fixed cost. In what follows, we will briefly discuss the conditions on the parameters under which optimal demand for services and the remanufacturability are positive. These conditions are useful in comparative statics and other analyses later on. The optimal w can be found by solving the first order condition (7) as shown in (10). From (9), the optimal w is a decreasing function of the optimal remanufacturability. This is intuitive as increased remanufacturability implies a decreased average variable cost of manufacturing/ remanufacturing per product.

$$w_m = \frac{\alpha\beta + \gamma C_m - R_m \gamma \Delta}{2\gamma} \qquad (10)$$

From (10) and $p(w)$ as given by (3), the optimal demand $\beta - \gamma p(w_m)$ is an increasing function of R_m as shown by (10).

$$\beta - \gamma p(w_m) = \frac{\alpha\beta - \gamma C_m + R_m\gamma\Delta}{4\alpha} \tag{11}$$

From (11), $\alpha\beta - \gamma C_m + R_m\gamma\Delta > 0$ if the optimal demand is positive. In addition,

substituting (10) into the first order condition (8), the rate of change of profit with R when

$R = 0$ and w is the optimal w given $R = 0$ is given by (12).

$$\frac{\partial\pi_m^m}{\partial R} = \Delta\left(\frac{\alpha\beta - \gamma C_m}{4\alpha^2}\right) - I'(0) \tag{12}$$

Given that π_m^m is concave, the condition for the optimal remanufacturability R_m to be positive

is $\frac{\partial\pi_m^m}{\partial R} > 0$ at $R = 0$. From (12), the conditions for R_m to be positive are that $\alpha\beta > \gamma C_m$ and

(13) is true.

$$\Delta\left(\frac{\alpha\beta - \gamma C_m}{4\alpha^2}\right) > I'(0) \tag{13}$$

From (13), we observe that conditions leading to a higher demand such as a high β and a low

γ are necessary for remanufacturing to be profitable. Given that an investment in

remanufacturability is a fixed cost, it is intuitive that a higher demand for services and

products is favorable to remanufacturing. Also note from Appendix B, that $\alpha\beta - \gamma C_m > 0$

if the optimal demand in the forward only supply chain is positive.

3.3 Comparative Statics of MDSC Equilibrium

In this section, we perform comparative statics to analyze how MDSC equilibrium R_m and w_m

vary with respect to key parameters such as $\gamma, \Delta,$ and $\alpha.$ Given the demand for products, the

parameter Δ signifies the extent of production cost savings via increased remanufacturing. Comparative statics with respect to Δ enables us identify how an increased incentive for remanufacturing impacts the optimal product life. Given the linear demand function $D(p) = \beta - \gamma p$, the marginal demand is $\frac{dD(p)}{dp} = -\gamma$. Thus increased γ implies that the demand is more sensitive to the price and may result in reduced demand for services overall. The product life α is inversely related to the number of products required to meet a given demand for services. A decrease in the number of products implies that the overall production cost savings due to remanufacturing for a given R and Δ would be less. We utilize the implicit function theorem to perform this analysis [32].

3.3.1 Comparative Statics with Respect to Marginal Demand γ

Equation(14) shows $D^2\pi_m^m$, the hessian matrix for π_m^m with respect to w and R.

$$D^2\pi_m^m = \begin{pmatrix} -\dfrac{\gamma}{\alpha^2} & -\dfrac{\gamma\Delta}{2\alpha^2} \\ -\dfrac{\gamma\Delta}{2\alpha^2} & -I''(R) \end{pmatrix} \tag{14}$$

The implicit function theorem is applicable if we have an interior equilibrium, i.e., the first order conditions are met and $|D^2\pi_m^m| \neq 0$. Here, $|D^2\pi_m^m|$ denotes the determinant of the hessian $D^2\pi_m^m$. From the implicit function theorem, we have,

$$\begin{pmatrix} \dfrac{\partial w_m}{\partial \gamma} \\ \dfrac{\partial R_m}{\partial \gamma} \end{pmatrix} = \begin{pmatrix} \dfrac{\partial^2\pi_m^m}{\partial w^2} & \dfrac{\partial^2\pi_m^m}{\partial w\partial R} \\ \dfrac{\partial^2\pi_m^m}{\partial R\partial w} & \dfrac{\partial^2\pi_m^m}{\partial R^2} \end{pmatrix}^{-1} \begin{pmatrix} -\dfrac{\partial^2\pi_m^m}{\partial w\partial \gamma} \\ -\dfrac{\partial^2\pi_m^m}{\partial R\partial \gamma} \end{pmatrix} \tag{15}$$

Where,

$$\begin{pmatrix} \dfrac{\partial^2 \pi_m^m}{\partial w \partial \gamma} \\ \dfrac{\partial^2 \pi_m^m}{\partial R \partial \gamma} \end{pmatrix} = \begin{pmatrix} \dfrac{1}{2\alpha} \\ -\dfrac{\Delta}{2\alpha} \end{pmatrix} \tag{16}$$

Substituting the hessian (14) and (16) into (15) and simplifying,

$$\frac{dR_m}{d\gamma} = -\frac{\Delta(C_m - R\Delta)}{4\alpha^2 I''(R_m) - \gamma\Delta^2} \tag{17}$$

Solving (15) and simplifying the resulting the expression by substituting w_m from (10)

is

$$\frac{dw_m}{d\gamma} = \frac{1}{2}\left(-\frac{\alpha\beta}{\gamma^2} + \frac{\Delta^2(C_m - R\Delta)}{4\alpha^2 I''(R_m) - \gamma\Delta^2}\right) \tag{18}$$

From (17) $\frac{dR_m}{d\gamma} < 0$. This is because increased γ results in a reduced demand for services and ultimately a lesser incentive to invest in remanufacturability. The sensitivity with respect to w_m depends on the specific parameter values and the investment function $I(R)$. Several investment functions such as $I(R) = kR^2$, $I(R) = Ae^{bR}$ and $I(R) = -\lambda \log(1-R)$ where k, A, b and λ are parameters satisfy $I'''(R) \geq 0$. Assuming that $I(R)$ satisfies $I'''(R) \geq 0$, from (18), when γ is sufficiently low, w_m decreases with γ as in the forward only channel. Conversely, when γ is sufficiently high, the w_m may increase with γ unlike the forward supply chain. To gain intuition, note that we can substitute (17) into (18) to obtain,

$$\frac{dw_m}{d\gamma} = \frac{1}{2}\left(-\frac{\alpha\beta}{\gamma^2} - \frac{dR_m}{d\gamma}\right) \tag{19}$$

Hence, when Δ is sufficiently high, there is a substantial decrease in remanufacturability with

γ representing an increase in the average cost of manufacturing/remanufacturing per product

which ultimately leads to increased wholesale price w. We can obtain an expression for $\frac{dp_m}{d\gamma}$

from the reaction $p(w)$ and $\frac{dw_m}{d\gamma}$ (18) as shown in (20). In contrast to the forward supply

chain (Appendix B), the price p_m increases with γ when γ is sufficiently high for similar

reasons why the wholesale price increases with γ.

$$\frac{dp_m}{d\gamma} = \frac{1}{2}\left(-\frac{3\beta}{2\gamma^2} + \frac{\Delta^2(C_m - R\Delta)}{2\alpha(4\alpha^2 I''(R_m) - \gamma\Delta^2)}\right) \tag{20}$$

3.3.2 Comparative Statics with Respect to Cost Saving per Remanufactured Product Δ

From the implicit function theorem we have,

$$\begin{pmatrix} \dfrac{\partial w_m}{\partial \Delta} \\ \dfrac{\partial R_m}{\partial \Delta} \end{pmatrix} = \begin{pmatrix} \dfrac{\partial^2 \pi_m^m}{\partial w^2} & \dfrac{\partial^2 \pi_m^m}{\partial w \partial R} \\ \dfrac{\partial^2 \pi_m^m}{\partial R \partial w} & \dfrac{\partial^2 \pi_m^m}{\partial R^2} \end{pmatrix}^{-1} \begin{pmatrix} -\dfrac{\partial^2 \pi_m^m}{\partial w \partial \Delta} \\ -\dfrac{\partial^2 \pi_m^m}{\partial R \partial \Delta} \end{pmatrix} \tag{21}$$

$$\begin{pmatrix} \dfrac{\partial^2 \pi_m^m}{\partial w \partial \Delta} \\ \dfrac{\partial^2 \pi_m^m}{\partial R \partial \Delta} \end{pmatrix} = \begin{pmatrix} \dfrac{1}{2\alpha} \\ -\dfrac{\Delta}{2\alpha} \end{pmatrix} \tag{22}$$

Substituting the hessian (14) and (22) into (21), we have

$$\frac{dR_m}{d\Delta} = \frac{2\alpha\beta - 2w\gamma + R\gamma\Delta}{4\alpha^2 I''(R_m) - \gamma\Delta^2} \tag{23}$$

Substituting (10) into (23),

$$\frac{dR_m}{d\Delta} = \frac{\alpha\beta - C_m\gamma + 2R\gamma\Delta}{4\alpha^2 I''(R_m) - \gamma\Delta^2} \tag{24}$$

Since $\alpha\beta > C_m$ from (24), as expected the optimal remanufacturability is increasing with Δ.

Equation (25) provides the resulting expression for $\frac{dw_m}{d\Delta}$ from the solution of (21).

$$\frac{dw_m}{d\Delta} = -\frac{\Delta\alpha\beta - \Delta w_m\gamma + 2R\alpha^2 I''(R_m)}{4\alpha^2 I''(R_m) - \gamma\Delta^2} \tag{25}$$

Substituting (10) into (25),

$$\frac{dw_m}{d\Delta} = -\frac{\Delta(\alpha\beta - C_m\gamma + R\gamma\Delta) + 4R\alpha^2 I''(R_m)}{2(4\alpha^2 I''(R_m) - \gamma\Delta^2)} \tag{26}$$

From (26), the equilibrium w always decreases with Δ. Also considering the reaction $p(w)$

we have that the equilibrium price p_m always decreases with Δ as well as shown in (27).

$$\frac{dp_m}{d\Delta} = -\frac{\Delta(\alpha\beta - C_m\gamma + R\gamma\Delta) + 4R\alpha^2 I''(R_m)}{4\alpha(4\alpha^2 I''(R_m) - \gamma\Delta^2)} \tag{27}$$

The results show that an increase in Δ results in a reduced w and p and an increase in R. A

possible rationale for this result is that given an increased Δ, w and p are reduced so that the

increased cost savings $R\Delta$ are realized from a greater number of products.

3.3.3 Comparative Statics with Respect to Product Life α

From implicit function theorem we have,

$$\begin{pmatrix} \dfrac{\partial w_m}{\partial \alpha} \\ \dfrac{\partial R_m}{\partial \alpha} \end{pmatrix} = \begin{pmatrix} \dfrac{\partial^2 \pi_m}{\partial w^2} & \dfrac{\partial^2 \pi_m}{\partial w\partial R} \\ \dfrac{\partial^2 \pi_m}{\partial R\partial w} & \dfrac{\partial^2 \pi_m}{\partial R^2} \end{pmatrix}^{-1} \begin{pmatrix} -\dfrac{\partial^2 \pi_m}{\partial w\partial \alpha} \\ -\dfrac{\partial^2 \pi_m}{\partial R\partial \alpha} \end{pmatrix} \tag{28}$$

$$\begin{pmatrix} \dfrac{\partial^2 \pi_m}{\partial w \partial \alpha} \\[2mm] \dfrac{\partial^2 \pi_m}{\partial R \partial \alpha} \end{pmatrix} = \begin{pmatrix} \dfrac{1}{2\alpha} \\[2mm] -\dfrac{\Delta}{2\alpha} \end{pmatrix} \tag{29}$$

Substituting the hessian (14) and (29) into (28), we have

$$\frac{dR_m}{d\alpha} = -\frac{\Delta \left(\beta - \dfrac{2C_m \gamma + 2R\gamma\Delta}{\alpha} \right)}{4\alpha^2 I''(R_m) - \gamma\Delta^2} \tag{30}$$

From (30) R_m decreases with α when α is sufficiently large i.e. when $\alpha > 2\gamma(C_m + \Delta)/\beta$.

Conversely, R_m increases with α when α is sufficiently small i.e when $\alpha < 2\gamma(C_m)/\beta$.

When $2\gamma(C_m + \Delta)/\beta \geq \alpha \geq 2\gamma C_m/\beta$, R_m can increase or decrease with α depending on the

investment function $I(R)$. An intuitive interpretation for this result is as follows. From the

first order condition (9), the optimal R_m is a function of the optimal demand for

products $(\beta - \gamma p(w_m))/\alpha$. Consequently, the sign of $\frac{dR_m}{d\alpha}$ depends on whether the optimal

demand for products increases or decreases with α. If the demand for services is held

constant then increased α implies that the demand for services can be met with a fewer

products. On the other hand increased α represents a decrease in the cost of providing the

services and may be an incentive for increasing the demand for services. When α is

sufficiently large, the increase in demand for services with α is sufficiently small such that

there is a decrease in the demand for products $(\beta - \gamma p)/\alpha$ and thus a decrease in R. When α

is sufficiently small, the increase in demand for services is large enough to cause an increase

in the demand for products and thus an increase in R.

Equation (31) provides an expression for $\frac{dw_m}{d\alpha}$ obtained by substituting (14) and (29) into (28) and solving,

$$\frac{dw_m}{d\alpha} = \frac{\gamma(\alpha\beta - 2w\gamma)\Delta^2 - (\alpha\beta + 2\gamma(C_m - R_m\Delta - 2w))2\alpha^2 I''(R)}{\alpha\gamma(4\alpha^2 I''(R_m) - \gamma\Delta^2)} \tag{31}$$

Substituting (10) into (31) and simplifying

$$\frac{dw_m}{d\alpha} = \frac{2\beta\alpha^3 I''(R_m) - \gamma\Delta^2(C_m\gamma - R_m\gamma\Delta)}{\alpha\gamma(4\alpha^2 I''(R_m) - \gamma\Delta^2)} \tag{32}$$

From (32), $\frac{dw_m}{d\alpha} > 0$ since $\alpha\beta > C_m\gamma$ and considering the assumption that $2\alpha^2 I''(R) > \gamma\Delta^2$.

The increase in w with α seems intuitive since more units of service can be provided per product with a greater α.. We can obtain an expression for $\frac{dp_m}{d\alpha}$ from (32) and the reaction function $p(w)$ as shown in

$$\frac{dp_m}{d\alpha} = \frac{\Delta^2(\alpha\beta - C_m\gamma + R\gamma\Delta) - 4(C_m - R\Delta)\alpha^2 I''(R_m)}{4\alpha^2(4\alpha^2 I''(R_m) - \gamma\Delta^2)} \tag{33}$$

3.4 Comparison of MDSC and CCSC

In this Section, we analyze how the lack of coordination in MDSC impacts the investment in remanufacturability and the price per unit service by comparing MDSC equilibrium with the benchmark CCSC solution.

3.4.1 The Centrally Coordinated Supply Chain (CCSC)

In this section, we assume that the manufacturer and the service provider are centrally coordinated by a central planner. Since financial transactions between the manufacturer and

the service provider do not directly influence the total SC profits, the relevant decision variables are the remanufacturability *R* and the price per unit service *p*. The total revenue for CCSC is given by *(β-γp) p* and the total cost for manufacturing/remanufacturing to provide the *(β-γp)* services is given by *(β-γp)(C_m-R Δ)/α*. The central planner's profit maximization problem is formulated by (34).

$$\underset{p,R}{Max}\ \ \pi_c = (\beta - \gamma p)p - \frac{(\beta - \gamma p)}{\alpha}(C_m - \Delta R) - I(R\ ) \tag{34}$$

The first derivatives of CCSC profit π_c with respect to *p* and *R* are provided by (35) and (36) respectively.

$$\frac{\partial \pi_c}{\partial p} = \beta - 2\gamma p + \frac{\gamma(C_m - R\Delta)}{\alpha} \tag{35}$$

$$\frac{\partial \pi_c}{\partial R} = \frac{(\beta - \gamma p)\Delta}{\alpha} - I'(R) \tag{36}$$

The Hessian matrix of CCSC profit π_c is given by (37)

$$\begin{pmatrix} -2\gamma & -\dfrac{\gamma\Delta}{\alpha} \\ -\dfrac{\gamma\Delta}{\alpha} & -I''(R_{sp}) \end{pmatrix} \tag{37}$$

The principal minors of the first order are clearly negative since *I(R)* is convex by assumption. Hence, the necessary condition for strict concavity is that the determinant of the Hessian is positive i.e.,

$$I''(R) - \frac{\gamma\Delta^2}{2\alpha^2} > 0 \tag{38}$$

Given that (38) is true, the optimal remanufacturability R_c and p_c are found by equating the first derivatives (34) and (35) to zero.

3.4.2 Comparison of Remanufacturabilities and Prices in MDSC and CCSC

In this section, we show that the optimal remanufacturability in MDSC R_{sp} is less than the optimal remanufacturability in CCSC R_c.

Proof: The first order condition (9) is satisfied at MDSC equilibrium. Considering (11), the first order condition (9) can be expressed as shown in (39).

$$\Delta\left(\frac{\alpha\beta - \gamma C_m + R_m\gamma\Delta}{4\alpha^2}\right) = I'(R_m) \tag{39}$$

Suppose, instead of the optimal R_c, we set the remanufacturability at $R = R_m$ in CCSC and then find the best price p given that $R = R_m$ by setting the first derivative (35) to zero,

$$p_{|R=R_m} = \frac{\alpha\beta + \gamma(C_m - R_m\Delta)}{2\gamma\alpha} \tag{40}$$

Then, the rate of change of profit π_c with increase in R at $R = R_m$ and $p_{|R=R_m}$ is given by equating the first derivative (36) to zero and substituting for $p_{|R=R_m}$ from (40).

$$\frac{\partial\pi_c}{\partial R_{R=R_m,P|R=R_m}} = \frac{\Delta(\alpha\beta - C_m\gamma + R_m\gamma\Delta)}{2\alpha^2} - I'(R_m) \tag{41}$$

Substituting the value of $I'(R_m)$ from (39) into (41),

$$\frac{\partial \pi_c}{\partial R_{R=R_m, p|R=R_m}} = \frac{\Delta(\alpha\beta - C_m\gamma + R_m\gamma\Delta)}{2\alpha^2} \tag{42}$$

Therefore, from (42) if $R = R_m$ in CCSC, we can increase the total profit π_c by increasing R.

However as π_c is a concave function of R and p, $R_c > R_m$. We will now show that the

optimal price in MDSC p_m is greater than the optimal price in CCSC p_c.

Proof: Considering (10), the optimal price in MDSC is

$$p_m = \frac{3\alpha\beta + C_m\gamma - R_m\gamma\Delta}{4\alpha\gamma} \tag{43}$$

Equating the first derivative (35) to zero,

$$p_c = \frac{\alpha\beta + C_m\gamma - R_c\gamma\Delta}{2\alpha\gamma} \tag{44}$$

Therefore,

$$p_m - p_c = \frac{\alpha\beta - C_m\gamma + (2R_c - R_m)\gamma\Delta}{4\alpha\gamma} \tag{45}$$

From (45), $p_m - p_c > 0$ since $R_c - R_m > 0$ and $\alpha\beta > C_m\gamma$. The above results show that the

lack of coordination (Double marginalization) leads to a higher price for the customers and

lower investment in remanufacturability when the manufacturer is the Stackelberg leader.

One interpretation of this proposition is that the double marginalization in the decentralized

MDSC leads to decreased demand and thus a lesser incentive to invest in remanufacturing.

3.5 Analysis with a Quadratic Investment Function

In this section, we will illustrate the analytical results derived previously with a specific quadratic investment function $I(R) = kR^2$ and numerical examples. The use of specific functional form enables us to gain additional insights, perform sensitivity analysis and obtain directions for further exploration. The quadratic investment function is often used to represent investment with diminishing results and results in closed form expressions for most optimal quantities. A similar investment function $I(q) = kq^2$ where q is the quality has been utilized by [27, 33].

Given this investment function, the parameter k represents the investment required for a remanufacturability of 1. In this case, the condition for concavity, Inequality (38) becomes

$2k > \dfrac{\gamma \Delta^2}{2\alpha^2}$. The optimal quantities can be solved for straight forwardly from the equilibrium

conditions. Table 1 provides the equilibrium R, p, demand and the total profits in MDSC and CCSC.

Table 1. Expressions for *p*, *R*, total supply profits and optimal demand in MDSC and CCSC

MDSC	CCSC
$R_m^* = \dfrac{(\alpha\beta - C_m\gamma)\Delta}{8k\alpha^2 - \gamma\Delta^2}$	$R_c^* = \dfrac{(\alpha\beta - C_m\gamma)\Delta}{4k\alpha^2 - \gamma\Delta^2}$
$p_m^* = \dfrac{2k\alpha(3\alpha\beta + C_m\gamma) - \beta\gamma\Delta^2}{\gamma(8k\alpha^2 - \gamma\Delta^2)}$	$P_c^* = \dfrac{2k\alpha(\alpha\beta + C_m\gamma) - \beta\gamma\Delta^2}{\gamma(4K\alpha^2 - \gamma\Delta^2)}$
$\pi_m^{total} =$ $\dfrac{k(\alpha\beta - C_m\gamma)^2 \left(12k\alpha^2 - \gamma\Delta^2\right)}{\left(8k\alpha^2 - \gamma\Delta^2\right)^2 \gamma}$	$\pi_c = \dfrac{(\alpha\beta - C_m\gamma)^2 k}{\left(4k\alpha^2 - \gamma\Delta^2\right)\gamma}$
$D(p_m^*) = \dfrac{2k\alpha(\alpha\beta - C_m\gamma)}{8k\alpha^2 - \gamma\Delta^2}$	$D(p_c^*) = \dfrac{2k\alpha(\alpha\beta - C_m\gamma)}{4k\alpha^2 - \gamma\Delta^2}$

3.5.1 Illustration of the Analytical Results

1) Remanufacturabilities: Equation (46) provides an expression for R_m/R_c. From (46), we

can observe that R_m becomes a smaller fraction of R_c as delta increases.

$$\frac{R_m}{R_c} = 1 - \frac{4k\alpha^2}{8k\alpha^2 - \gamma\Delta^2} \qquad (46)$$

2) Prices: Equation (47) provides an expression for $p_m - p_c$. From (47) $p_m > p_c$ since

$\alpha\beta - C_m\gamma > 0$ if demand is positive in any SC.

$$p_m - p_c = \frac{k\alpha(\alpha\beta - C_m\gamma)\Delta^2}{(4k\alpha^2 - \gamma\Delta^2)(8k\alpha^2 - \gamma\Delta^2)} \tag{47}$$

3.5.2 Additional Observations from $I(R) = kR^2$

Percentage profit gain with coordination ζ: The percentage ζ is a measure of the benefit

obtained by coordinating a decentralized supply chain and is defined as

$100(\pi_{coordinated} - \pi_{decentr\,alized})/\pi_{mdecentralized}$. Equation (48) provides an expression ζ

of MDSC. From (48), ζ for MDSC is greater than 33%.

$$\xi_{MDSC} = 100 \frac{16k^2\alpha^4}{\left(4k\alpha^2 - \gamma\Delta^2\right)\left(12k\alpha^2 - \gamma\Delta^2\right)} \tag{48}$$

From (48), ζ_{MDSC} increases with increase in marginal demand γ or the cost savings due to

remanufacturing Δ since $4k\alpha^2 > \gamma\Delta^2$ if CCSC profit is concave. In general, this result

indicates that conditions that are conducive to remanufacturing such as a high Δ, low k and α

$($a lower product life under some conditions can lead to a lower remanufacturability$)$ also

result in a higher percent gain with coordination in MDSC

3.5.2 Numerical Examples

We will now present a couple of numerical examples with hypothetical data to illustrate the

results derived with $I(R) = kR^2$. The parameters of the first example are such that the optimal

R in CCSC is less than 0.65. The specific parameter values for the first example are $k = 1$

000 000, $C_m = 23737$, $\beta = 8\,000\,000$ and $\gamma = 2\,500\,000$. We selected the parameters α and Δ

for sensitivity analysis. We first varied Δ between 2000 and 7000 with a fixed α of 12 113.

In this case, the central planner's profit would be concave and we would have a unique

Stackelberg equilibrium in MDSC if $\Delta < 15321$. Figure 2 and Figure 3 show the variation of

R and p with Δ respectively in MDSC and CCSC. From Figure 2, we can observe that R is

nonlinearly increasing with Δ. In addition, the difference between the remanufacturabilities

of CCSC and MDSC is the greatest when Δ is large. This observation can be expected from

(46) and can be interpreted as follows. From Table 1, the optimal R is equal to the optimal

demand multiplied by $\Delta/2k\alpha$. The optimal demand is lesser in MDSC compared to CCSC

due to double marginalization implying that the additional cost savings due an increase in Δ

would be applicable to a lesser number of products resulting in a lesser incentive to increase

the remanufacturability. Furthermore, from Table 1, the optimal demand increases at a

greater rate with Δ in MDSC compared to CCSC. Similarly, the difference between the price

p in CCSC and SPDSC is the greatest when Δ is large.
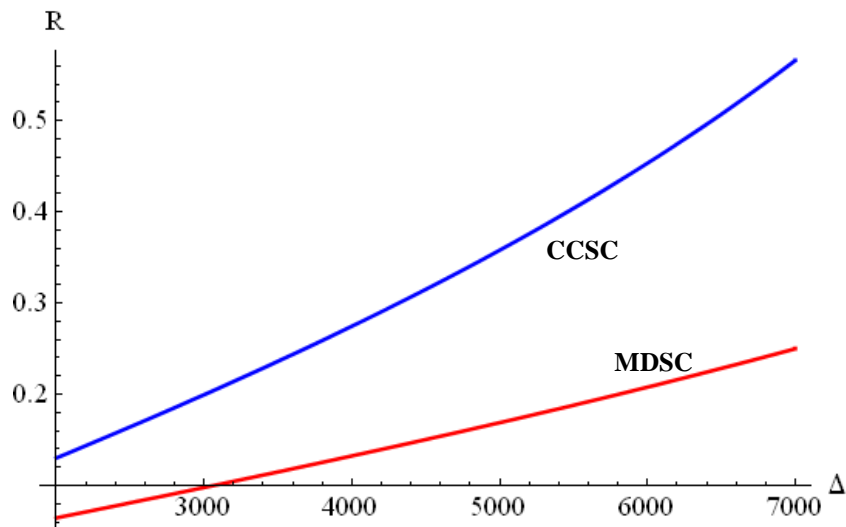
Figure 2 Variation of R with Δ in MDSC and CCSC
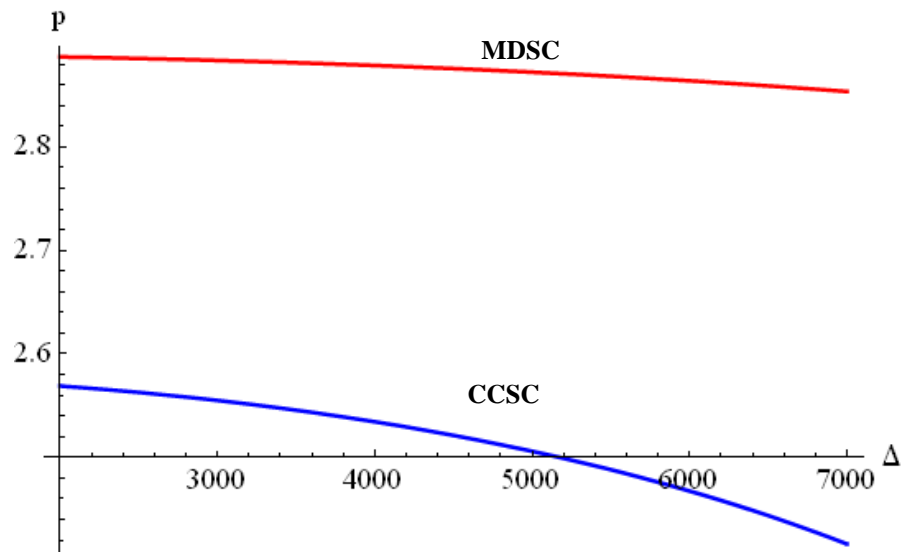
Figure 3 Variation of *p* with Δ in MDSC and CCSC

From Figure 4, we observe that the percentage profit gain with coordination ζ is nonlinearly
increasing with Δ for MDSC. As we observed, CCSC solution is more responsive to
increased cost savings per remanufactured product with a greater increase in
remanufacturability and decrease in the price compared to MDSC. Hence, coordination is
much more beneficial when Δ is high given a quadratic investment function.
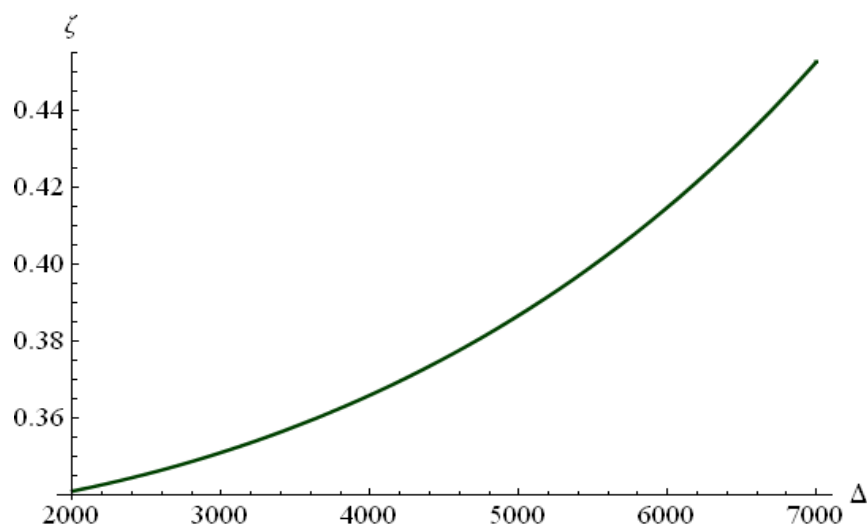
Figure 4 Variation of the Percentage Gain ζ in MDSC with Δ

To observe the sensitivity with respect to the product life, we varied α between 8000 and 20 000 while fixing Δ at 6500. and show the variation of R and p in MDSC and CCSC with α while Δ is 6500. We can observe that at higher values of α, the optimal R is decreasing with α in MDSC as well as CCSC. In this case, a higher α is leading to a decrease in the overall demand for products $(\beta - \gamma p)/\alpha$ implying a lesser incentive for fixed cost investment in R. Counter intuitively R increases with α when α is sufficiently small. In this case, an increase in product life is leading to a sufficiently large increase in the demand for services $(\beta - \gamma p)$ such that the demand for products $(\beta - \gamma p)/\alpha$ is increasing with α.
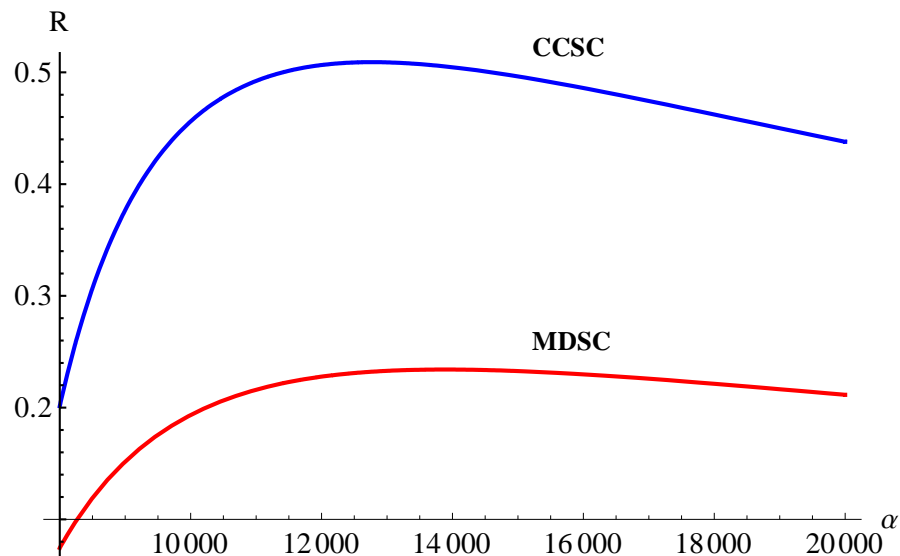
Figure 5 Variation of R with α in MDSC and CCSC
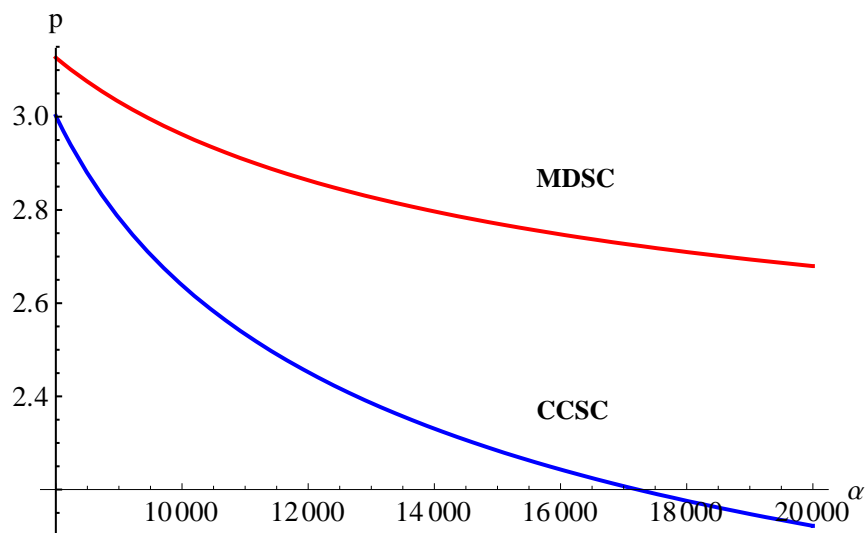
Figure 6 Variation of p in MDSC and CCSC with α
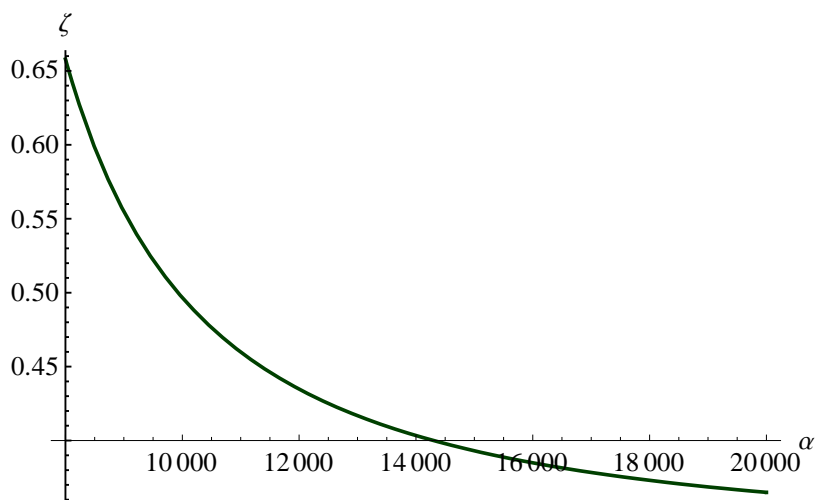
Figure 7 Variation of the Percentage Profit Gain ζ in MDSC with α

4. The Manufacturer Driven Supply Chain (MDSC) with Variable Product Life

A longer product life and increased remanufacturing have both been recommended from an environmental perspective. Meanwhile, some of the qualitative reports have suggested that selling services instead of selling goods can lead to increased product reuse and a greater product life. In this chapter, we extend MDSC to consider investment in remanufacturability as well as variable product life α and investigate under what conditions is a higher remanufacturability or a greater product life preferred. Specifically, we assume that the product life can be increased by utilizing longer life components that result in a greater variable cost of manufacturing C_m.

This chapter is organized as follows. In section 4.1 we introduce the additional notation related to the variable product life and discuss the modeling assumptions. Next in section 4.2, we formulate MDSC with variable product life as a Stackelberg game and derive the equilibrium conditions. In Section 4.3, we perform comparative statics analysis to analyze how γ and Δ impact the equilibrium product life and remanufacturability. Next in Section 4.4, we compare the α, R and p between MDSC and CCSC. Finally, we provide numerical examples in Section 4.5.

4.1 Notation and Assumptions

As discussed in the introduction, we assume that the product life α can be increased by utilizing better quality components resulting in a higher variable cost of manufacturing (e.g, copier bushings with better thermal properties[17]). Hence, we assume that the manufacturing cost per product C_m is a convex and increasing function of product life $C_m(\alpha)$.

Similarly, we assume that the remanufacturing cost per product C_r is a convex and increasing function of product life $C_r(\alpha)$. We note that similar functions relating the manufacturing cost to the product life have been utilized in other relevant articles [18, 19].

We assume a constant cost savings per remanufactured product regardless of the product life, i.e., $C_m(\alpha) - C_r(\alpha) = \Delta \ \forall \ \alpha$. The assumption is justified as follows. A product is composed of multiple components which vary with respect to their component life. In a typical remanufacturing process, the production cost savings Δ is achieved via the reuse of relatively long life components also referred to "core components" (e.g, electric motors in the case of copiers and casing for tires) [34]. Meanwhile, the product life is most impacted by the life of relatively short life components that are replaced during the remanufacturing process. Hence, the cost savings per remanufactured product Δ would not be significantly affected if the product life is improved via utilizing better quality short life components that are replaced in the remanufacturing process.

We assume that the product life values of interest are greater than a lower limit α_l. The lower limit is based on the grounds that there is often some technological limit beyond which it is not feasible to further reduce the product life.

4.2 Stackelberg Game Formulation

As in the case when product life was a parameter, we model this supply chain as a Stackelberg game where the manufacturer is the leader and makes his decisions first while the service provider is the follower and makes her decisions later. Since the service provider is the follower in our MDSC, she accepts the price per product w and the product life α that is

determined by the manufacturer, and in turn determines the price per unit service p that maximizes her profits. Being the leader, the manufacturer anticipates the service provider's best response function $p(w, \alpha)$ and determines the optimal wholesale price w_m, the remanufacturability R_m and the product life α_m that maximize his profit. In what follows, we will derive sufficient conditions for the existence and uniqueness of the Stackelberg equilibrium and state the equilibrium conditions.

To solve the Stackelberg game we first optimize the service provider's profit and determine her best response function $p(w, \alpha)$ to a given w and α. The best response function refers to the optimal price p that maximizes the service provider's profit π_{sp}^m given the wholesale price w and the product life α. The service provider's profit maximization is formulated by (49). The term $(\beta - \gamma p)$ in (49) is the demand for services while $(p - \frac{w}{\alpha})$ is the service provider's margin per unit service.

$$\begin{array}{c} Max \\ p \end{array} \pi_{sp}^m = (\beta - \gamma p)(p - \frac{w}{\alpha}) \qquad (49)$$

In the Stackelberg game, concavity of the follower's objective implies that his best response function is single valued and is a sufficient condition for the existence of the Stackelberg equilibrium[31]. From (49), $\frac{\partial^2 \pi_{sp}^m}{\partial p^2} = -2\gamma < 0$. Therefore, π_{sp}^m is a concave function of the price p implying that the service provider's best response function is single valued. The first order condition for maximizing the service provider's profit is given by (50).

$$\frac{\partial \pi_{sp}^m}{\partial p} = \beta - 2\gamma p + \frac{w}{\alpha} = 0 \qquad (50)$$

Solving (50) the service provider's best response function is as provided by (51).

$$p(w, \alpha) = \frac{\alpha\beta + w\gamma}{2\alpha\gamma} \tag{51}$$

The next step in solving the Stackelberg game is to determine the w, R and α that maximize the manufacturers profit π_m^m while considering the service provider's best response function $p(w, \alpha)$. The manufacturer's profit maximization problem is formulated by (52). Here, the term $(\beta - \gamma p(w, \alpha))/\alpha$ is the total number of products while $(w - C_m(\alpha) + R\Delta)$ is the manufacturer's margin per product. The fixed cost investment required to achieve a remanufacturability of R is represented by $I(R)$.

$$\underset{w, R, \alpha}{Max} \; \pi_m^m = \frac{(\beta - \gamma p(w, \alpha))}{\alpha}(w - C_m(\alpha) + R\Delta) - I(R) \tag{52}$$

Substituting the service provider's best response function $p(w, \alpha)$ from (51) into (52),

$$\pi_m^m = \frac{\alpha\beta - w\gamma}{2\alpha^2}(w - C_m(\alpha) + R\Delta) - I(R) \tag{53}$$

The first order conditions for maximizing π_m^m are given by (54), (55) and (56).

$$\frac{\partial \pi_m^m}{\partial w} = \frac{\alpha\beta - 2w\gamma - R\gamma\Delta + \gamma C_m(\alpha)}{2\alpha^2} = 0 \tag{54}$$

$$\frac{\partial \pi_m^m}{\partial R} = \frac{\Delta(\alpha\beta - w\gamma)}{2\alpha^2} - I'(R) = 0 \tag{55}$$

$$\frac{\partial \pi_m^m}{\partial \alpha} = -\frac{(\alpha\beta - 2w\gamma)(w + R\Delta - C_m(\alpha)) + \alpha(\alpha\beta - w\gamma)C_m'(\alpha)}{2\alpha^3} = 0 \tag{56}$$

A sufficient condition for the existence of the Stackelberg equilibrium is the quasiconcavity of the leader's objective. A function f defined on a convex set U in R^n is quasiconcave for all $x, y \in U$ we have $f(x) \geq f(y)$ implies $f(\lambda x + (1 - \lambda) y) > f(y)$[32]. A sufficient condition for the quasi-concavity of π_m^m is that it is strictly concave at any critical point.

Here, a critical point is defined as any point where the first order conditions are met (See Ray *et al* [35] for a similar approach to prove the uniqueness of the Stackelberg equilibrium). Let $D^2\pi_m^m$ denote the hessian matrix of the manufacturer's profit as given by (57). We will have a unique Stackelberg equilibrium if the hessian $D^2\pi_m^m$ is negative definite at any critical point.

$$D^2\pi_m^m = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix}$$

Where

$$h_{11} = \frac{\partial^2 \pi_m^m}{\partial w^2} = -\frac{\gamma}{\alpha^2}$$

$$h_{12} = \frac{\partial^2 \pi_m^m}{\partial w \partial R} = -\frac{\gamma\Delta}{2\alpha^2}$$

$$h_{13} = \frac{\partial^2 \pi_m^m}{\partial w \partial \alpha} = \frac{\gamma(R\Delta - C_m(\alpha) + \alpha C_m'(\alpha))}{\alpha^2} \tag{57}$$

$$h_{22} = \frac{\partial^2 \pi_m^m}{\partial R^2} = -I''(R)$$

$$h_{23} = \frac{\partial^2 \pi_m^m}{\partial R \partial \alpha} = -\frac{\alpha\beta\Delta - 2w\gamma\Delta}{2\alpha^3}$$

$$h_{33} = \frac{\partial^2 \pi_m^m}{\partial \alpha^2} = \frac{1}{2\alpha^4}(2w\alpha\beta - 6w^2\gamma + 2R\alpha\beta\Delta - 6Rw\gamma\Delta + (6w\gamma - 2\alpha\beta)C_m(\alpha)$$
$$+ 2\alpha(\alpha\beta - 2w\gamma)C_m'(\alpha) + \alpha^2(w\gamma - \alpha\beta)C_m''(\alpha))$$

The conditions for the hessian $D^2\pi_m^m$ provided by (57) to be negetive definete are 1) $h_{11}<0$, 2) $h_{11} h_{22} -(h_{12})^2>0$, and 3) Determinanant of the hessian $D^2\pi_m^m$ from (57) is negetive. Clearly, $h_{11} <0$. The second condition $h_{11} h_{22} -(h_{12})^2>0$ is elaborated in Inequalities (58) and (59) and will be met if $I''(R)$ is sufficiently large or product life α is sufficiently large.

$$h_{11}h_{22} - (h_{12})^2 = \frac{\gamma I''(R)}{\alpha^2} - \frac{\gamma^2\Delta^2}{4\alpha^4} > 0 \tag{58}$$

Simplifying inequality (58),

$$4\alpha^2 I''(R^*) > \gamma\Delta^2 \tag{59}$$

Let us now consider the final condition for π_m^m to be strictly concave at a critical point i.e. the $|D^2\pi_m^m|$ defined as the determinant of the hessian $D^2\pi_m^m$ from (57) is negetive. Equation (60) provides the expression for the determinant of $|D^2\pi_m^m|$.

$$
\begin{aligned}
|D^2\pi_m^m| \quad &= \frac{1}{8\alpha^8}(2\gamma^2(\alpha\beta - w\gamma)\Delta^2(w + R\Delta) + 8\alpha^2\gamma^2 C_m(\alpha)^2 I''(R) \\
&\quad - 2\gamma^2 C_m(\alpha)((\alpha\beta - w\gamma)\Delta^2 \\
&\quad + 4\alpha^2\left(w + 2R\Delta + \alpha C_m'(\alpha)\right)I''(R) \\
&\quad + \alpha^2(2\left((\alpha\beta - w\gamma)^2 + 4Rw\gamma^2\Delta + 4R^2\gamma^2\Delta^2\right. \\
&\quad + \alpha\gamma C_m'(\alpha)\left(2\alpha\beta + 4R\gamma\Delta + \alpha\gamma C_m'(\alpha)\right)I''(R) \\
&\quad + \gamma(\alpha\beta - w\gamma)C_m''(\alpha)\left(\gamma\Delta^2 - 4\alpha^2 I''(R)\right))))
\end{aligned}
\tag{60}
$$

However, since we only need to prove strict concavity at any critical point, we can simplify the expression for determinant of$|D^2\pi_m^m|$ shown in (60) by substituting the first order conditions (56) and (54). The resulting conditon for the determinant $|D^2\pi_m^m|$ to be negetive is shown in (61). Inequality (61) will be true if the maximum demand β is sufficiently low.

$$\Delta^2\beta - \gamma\Delta^2 C_m'(\alpha) - \alpha C_m''(\alpha)(4\alpha^2 I''(R) - \gamma\Delta^2)$$
$$< 0 \tag{61}$$

In summary, we will have a unique Stackelberg equilibrium if (61) and (59) are true in the relavent range of variables i.e., $0 < R < 1$ and $\alpha_l < \alpha$. The equilibrium wholesale price w_m, remanufacturability R_m and the product life α_m are implicitly determined by the first order conditions (54), (55) and (56). Now, since $\frac{\partial^2 \pi_m^m}{\partial w^2} < -\frac{\gamma}{\alpha^2}$, π_m^m is concave with respect to w, we can solve the first order condition (54) to find $\widehat{w}(R, \alpha)$, the optimal w for a given R and α (See Porteus [30] for a similar approach).

$$\widehat{w}(R, \alpha) = \frac{\alpha\beta + \gamma C_m(\alpha) - R\gamma\Delta}{2\gamma} \tag{62}$$

From (62), $\widehat{w}(R, \alpha)$ decreases with R when α is held constant because an increase in R represents a decrease in the average manufacturing/remanufacturing cost per product $C_m(\alpha) - R\Delta$. Conversely, $\widehat{w}(R, \alpha)$ increases with α given a constant R since an increase in α implies that a greater number of services can be provided from a single product. The term $(\alpha\beta - w\gamma)/2\alpha^2$ from the first order condition (55) is the optimal demand for products based on the service provider's best response function. This dependence of R on the optimal demand for products is possibly due to considering fixed cost investment in remanufacturability. Since both (54) and (56) are satisfied at the equilibrium, we can substitute (54) in (56) to obtain,

$$\frac{\partial \pi_m^m}{\partial \alpha} = -\frac{(\alpha_m\beta + \gamma C_m(\alpha_m) - R_m\gamma\Delta)(R_m\Delta - C_m(\alpha_m) + \alpha_m C_m'(\alpha_m))}{2\gamma} = 0 \tag{63}$$

Now, given the expression $\widehat{w}(R, \alpha)$ from (62) and the service provider's best response function $p(\mathrm{w}, \alpha)$, the equilibrium demand for service is related to the equilibrium α_m and R_m as shown by (64).

$$\beta - \gamma p(\widehat{w}(R_m, \alpha_m), \alpha_m) = \frac{\alpha_m \beta - \gamma C_m(\alpha_m) + \gamma R_m \Delta}{4\alpha_m} \qquad (64)$$

From (64), $\alpha_m \beta - \gamma C_m(\alpha_m) + \gamma R_m \Delta > 0$ if the optimal demand is positive. Therefore, from (63) we have that at the Stackelberg equilibrium $R\Delta - C_m(\alpha) + \alpha C_m'(\alpha) = 0$. The result shows that demand parameters do not directly affect the optimal product life except by their indirect effect via the optimal remanufacturability. This result may be due to modeling α as being associated with the variable cost of manufacturing and is consistent with Swan[19].

Let us examine what kind of manufacturing cost functions $C_m(\alpha)$ can satisfy the quasi concavity condition (61) as well as the first order condition $R\Delta - C_m(\alpha) + \alpha C_m'(\alpha)$. From (64), $\alpha_m \beta - \gamma C_m(\alpha_m) + \gamma R_m \Delta > 0$. Considering $R\Delta = C_m(\alpha) - \alpha C_m'(\alpha)$, $\alpha \beta > \gamma(C_m(\alpha) - C_m(\alpha) + \alpha C_m'(\alpha))$ i.e $\beta > \gamma C_m'(\alpha)$. Hence the term, $\Delta^2 \beta - \gamma \Delta^2 C_m'(\alpha)$ in quasiconcavity condition (61) is positive. Hence, π_m is quasiconcave if both $I''(R)$ and $C_m''(\alpha)$ are sufficiently large. Also, if the quasi concavity conditions are satisfied for all R, $0<R<1$, for a specific value of $\alpha = \alpha_l$, suppose $C_m'''(\alpha) \geq 0$, then the quasi concavity condition is satisfied for all $\alpha > \alpha_l$. Examples of investment functions that meet the above criteria are $C_m(\alpha) = C_{m0} + s\alpha^n$ and $C_m(\alpha) = C_{m0} + se^\alpha$ etc.

Let us now examine how the optimal remanufacturability impacts the optimal product life. Let $C_m(\alpha) - \alpha C_m'(\alpha) - R\Delta = G(\alpha, R) = 0$. Now, since $G(\alpha, R)$ is always satisfied at the Stackelberg equilibrium, we can utilize the implicit function to analyze how the optimal

remanufacturability influences the product life as shown in (65). From (65), we observe that the optimal product life decreases with an increase in the optimal remanufacturability as long as the functional form of $C_m(\alpha)$ and Δ do not change. The result highlights the conflict between the optimal product life and the optimal remanufacturability.

$$\frac{d\alpha}{dR} = \frac{-\frac{\partial G}{\partial R}}{\frac{\partial G}{\partial \alpha}} = -\frac{\Delta}{\alpha C_m''(\alpha\ )} < 0 \tag{65}$$

In what follows, we will perform comparative statics analysis to analyze how the equilibrium values vary with changes in key parameters.

4.3 Comparative Statics of the Stackelberg Equilibrium.

In this section, we will analyze how the equilibrium values R_m and α_m vary with changes in γ and Δ. We will convert the three variable optimization problem of maximizing π_m^m into an equivalent two variable form for ease of presentation(See Porteus [30] for a similar approach). Recall that since $\frac{\partial^2 \pi_m^m}{\partial w^2} < -\frac{\gamma}{\alpha^2}$, π_m^m is concave with respect to *w* and we can obtain an expression for the optimal wholesale price $\widehat{w}(R, \alpha)$ for a given R and α as provided by (66). We can substitute (66) into (52) to obtain an expression for $\pi_m^m\ (R,\ \alpha)$ defined as maximum manufacturer's profit by optimizing the wholesale price *w* for a given R and α as shown in (67).

$$\widehat{w}(R, \alpha) = \frac{\alpha\beta + \gamma C_m(\alpha) - R\gamma\Delta}{2\gamma} \tag{66}$$

$$\pi_m(\alpha, R) = \pi_m(\widehat{w}(R, \alpha), R, \alpha) = \frac{(\alpha\beta - \gamma C_m(\alpha) + \gamma R\Delta)^2}{8\alpha^2\gamma} - I(R) \tag{67}$$

The results of the comparative statics with respect to γ and Δ are summarized as Proposition 1.

PROPOSITION 1. *An increase in cost savings per remanufactured product Δ or a decrease in the marginal demand γ lead to increased demand for services, increased investment in remanufacturability and reduced product life in MDSC Stackelberg Equilibrium.*

Proposition 1 shows that conditions leading to a higher remanufacturability are also resulting in a shorter product life in MDSC. An interpretation of this result is that increased remanufacturability implies a decrease in the average variable cost of manufacturing/remanufacturing the product which is an incentive for decreasing the product life. The managerial implication is that manufacturers/remanufacturers should consider reducing the product life under the above conditions that are favorable for remanufacturing. The proof of proposition 1 is provided by the comparative statics in Sections 4.3.1 and 4.3.2.

4.3.1 Comparative Statics with Respect to Marginal Demand γ

We utilize the implicit function theorm to perform comparative statics with respect to γ [32]. Equation (68) provides $D^2\pi_m^m (R, \alpha)$ defined as the hessian matrix for $\pi_m^m (R, \alpha)$.

$$D^2\pi_m^m (R, \alpha) = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}$$

Where,

$$h_{11} = \frac{\partial^2 \pi_m^m(R, \alpha)}{\partial R^2} = \frac{\gamma\Delta^2}{4\alpha^2} - I''(R) \tag{68}$$

$$h_{12} = h_{21} = \frac{\partial^2 \pi_m^m(R,\alpha)}{\partial R \partial \alpha} = -\frac{\Delta(\alpha\beta - 2C_m(\alpha)\gamma + \alpha\gamma C_m'(\alpha) + 2R\gamma\Delta)}{4\alpha^3}$$

$$h_{22} = \frac{\partial^2 \pi_m^m(R,\alpha)}{\partial \alpha^2}$$

$$= \frac{1}{4\alpha^4}(R\Delta - C_m(\alpha)$$

$$+ \alpha\big(C_m'(\alpha)\big)\big(2\alpha\beta + 3R\gamma\Delta - 3\gamma C_m(\alpha) + \alpha\gamma C_m'(\alpha)\big) - \alpha^2 C_m''(\alpha)(\alpha\beta$$

$$+ R\gamma\Delta - \gamma C_m(\alpha))$$

The implicit function theorem is applicable if we have an interior equilibrium i.e., the first

order conditions are met and the hessian matrix $D^2\pi_m^m(R,\alpha)$ as provided by (68) is

nonsingular. Given the assumption that inequalities (59) and (61) are true, $\pi_m^m(R,\alpha)$ is

concave at the critical point and hence we can apply the implicit function theorem to

characterize the equilibrium. From implicit function theorem we have,

$$\begin{pmatrix} \dfrac{\partial R_m}{\partial \gamma} \\ \dfrac{\partial \alpha_m}{\partial \gamma} \end{pmatrix} = \begin{pmatrix} \dfrac{\partial^2 \pi_m(\alpha,R)}{\partial R^2} & \dfrac{\partial^2 \pi_m(\alpha,R)}{\partial R \partial \alpha} \\ \dfrac{\partial^2 \pi_m(\alpha,R)}{\partial R \partial \alpha} & \dfrac{\partial^2 \pi_m(\alpha,R)}{\partial \alpha^2} \end{pmatrix}^{-1} \begin{pmatrix} -\dfrac{\partial^2 \pi_m(\alpha,R)}{\partial R \partial \gamma} \\ -\dfrac{\partial^2 \pi_m(\alpha,R)}{\partial \alpha \partial \gamma} \end{pmatrix} \quad (69)$$

Substituting the hessian $D^2\pi_m^m(R,\alpha)$ provided by (68) into (69) and solving,

$$\begin{pmatrix} \dfrac{dR_m}{\partial \gamma} \\ \dfrac{d\alpha^*}{\partial \gamma} \end{pmatrix} = \begin{pmatrix} \dfrac{h_{12}\dfrac{\partial^2 \pi_m(\alpha,R)}{\partial \alpha \partial \gamma} - h_{22}\dfrac{\partial^2 \pi_m(\alpha,R)}{\partial R \partial \gamma}}{h_{11}h_{22} - h_{12}^2} \\ \dfrac{h_{21}\dfrac{\partial^2 \pi_m(\alpha,R)}{\partial R \partial \gamma} - h_{11}\dfrac{\partial^2 \pi_m(\alpha,R)}{\partial \alpha \partial \gamma}}{h_{11}h_{22} - h_{12}^2} \end{pmatrix} \quad (70)$$

From $\pi_m^m(R,\alpha)$ as provided by (67),

$$\begin{pmatrix} \dfrac{\partial^2 \pi_m(\alpha, R_m)}{\partial R \partial \beta} \\ \dfrac{\partial^2 \pi_m(\alpha, R_m)}{\partial \alpha \partial \beta} \end{pmatrix} = \begin{pmatrix} \dfrac{\Delta(C_m(\alpha) - R\Delta)}{4\alpha^2} \\ \dfrac{(C_m(\alpha) - R\Delta)(C_m(\alpha) - R\Delta - \alpha C_m'(\alpha))}{4\alpha^3} \end{pmatrix} \quad (71)$$

Substituting (71) and (68) into (70) we can obtain expressions for $\frac{\partial R_m}{\partial \gamma}$ and $\frac{\partial \alpha_m}{\partial \gamma}$ as given by

(72) and (73).

$$\frac{\partial R_m}{\partial \gamma}$$

$$= -\frac{\Delta(\alpha\beta - \gamma C_m(\alpha) + R\gamma\Delta)(\alpha^2 C_m''(\alpha) + C_m(\alpha) - \alpha C_m'(\alpha) - R\Delta)(C_m(\alpha) - R\Delta)}{16\alpha^6(h_{11}h_{22} - h_{12}^2)} \quad (72)$$

The term $\alpha\beta - \gamma C_m(\alpha) + R\gamma\Delta$ in (72) is positive if the optimal demand is positive. $C_m(\alpha) - R\Delta$ is positive since the average manufacturing cost per product is positive. $\alpha^2 C_m''(\alpha) + C_m(\alpha) - \alpha C_m'(\alpha) - R\Delta$ is positive since $C_m(\alpha) - \alpha C_m'(\alpha) - R\Delta = 0$ and $C_m''(\alpha) > 0$ by assumption. Finally, since $(h_{11}h_{22} - h_{12}^2) > 0$, $\frac{\partial R_m^*}{\partial \gamma}$ from (72) is negative.

$$\frac{d\alpha_m}{d\gamma}$$

$$= \frac{(\Delta^2(\alpha\beta - \gamma C_m(\alpha) + R\gamma\Delta) + 4\alpha^2 I''(R)(\alpha C_m'(\alpha) - C_m(\alpha) + R\Delta))(C_m(\alpha) - R\Delta)}{16\alpha^6(h_{11}h_{22} - h_{12}^2)} \quad (73)$$

The term $\alpha\beta - \gamma C_m(\alpha) + R\gamma\Delta$ in (73) is positive if the optimal demand is greater than zero. $C_m(\alpha) - R\Delta$ is positive since the average manufacturing per product is positive. $-(C_m(\alpha) - \alpha C_m'(\alpha) - R\Delta)$ is zero considering the first order condition(63). Finally, since $(h_{11}h_{22} - h_{12}^2) > 0$, $\frac{\partial \alpha_m}{\partial \gamma}$ is positive. We will now analyze how the equilibrium demand for services

varies with an increase in γ. From (64), the equilibrium demand for services is related to α_m and R_m as shown below.

$$\beta - \gamma p(\widehat{w}(R_m, \alpha_m), \alpha_m) = \frac{\beta}{4} - \frac{\gamma(C_m(\alpha_m) - R_m\Delta)}{4\alpha_m} \tag{74}$$

To analyze how an increase in γ impacts the optimal demand for services, we can differentiate the expression for $\beta - \gamma p(\widehat{w}(R_m, \alpha_m), \alpha_m)$ from (74) with respect to γ while noting that α_m and R_m should be considered as functions of γ as shown in (75).

$$\frac{d(\beta - \gamma p(\widehat{w}(R_m, \alpha_m), \alpha_m))}{d\gamma}$$
$$= \frac{\alpha_m\gamma(-C_m(\alpha_m) + R_m\Delta + \gamma\frac{dR_m}{d\gamma}) + \frac{d\alpha_m}{d\gamma}\gamma(\alpha C_m'(\alpha) - C_m(\alpha) + R\Delta))}{4\alpha_m} \tag{75}$$

From (75), the optimal demand for services decreases with an increase in γ, considering that $\alpha C_m'(\alpha) - C_m(\alpha) + R\Delta = 0$, $C_m(\alpha_m) > R_m\Delta$ and $\frac{dR_m}{d\gamma} < 0$ from (72). The preceding results show that when the marginal demand γ increases then it is beneficial to decrease the remanufacturability and increase the product life. An intuitive interpretation of this result is that an increase in γ results in decreased demand and hence a lesser incentive to invest in remanufacturability. The decreased remanufacturability leads to an increased average cost of manufacturing/remanufacturing per product which acts as an incentive for increasing the product life.

4.3.2 Comparative Statics with Respect to Cost Saving per Remanufactured Product Δ

We applied the Implicit function theorem to perform comparative statics analysis of MDSC

equilibrium with respect to Δ. Given that $h_{11}, h_{12}, h_{13},$ and h_{14} are elements of the hessian

matrix $D^2\pi_m^m(R, \alpha)$ as given by (68), from the implicit function theorem,

$$\begin{pmatrix} \dfrac{dR_m}{\partial\Delta} \\ \dfrac{d\alpha_m}{\partial\Delta} \end{pmatrix} = \begin{pmatrix} \dfrac{h_{12}\dfrac{\partial^2\pi_m(\alpha,R)}{\partial\alpha\partial\Delta} - h_{22}\dfrac{\partial^2\pi_m(\alpha,R)}{\partial R\partial\Delta}}{h_{11}h_{22} - h_{12}^2} \\ \dfrac{h_{21}\dfrac{\partial^2\pi_m(\alpha,R)}{\partial R\partial\Delta} - h_{11}\dfrac{\partial^2\pi_m(\alpha,R)}{\partial\alpha\partial\Delta}}{h_{11}h_{22} - h_{12}^2} \end{pmatrix} \tag{76}$$

From $\pi_m^m(R, \alpha)$ as provided by (67),

$$\begin{pmatrix} \dfrac{\partial^2\pi_m(\alpha,R)}{\partial R\partial\Delta} \\ \dfrac{\partial^2\pi_m(\alpha,R)}{\partial\alpha\partial\Delta} \end{pmatrix} = \begin{pmatrix} \dfrac{\Delta(\alpha\beta - \gamma C_m(\alpha) + 2R\gamma\Delta)}{4\alpha^2} \\ -\dfrac{(R)(\alpha\beta + 2R\gamma\Delta - 2\gamma C_m(\alpha) + \alpha\gamma C_m'(\alpha)}{4\alpha^3} \end{pmatrix} \tag{77}$$

We can obtain an expression for $\frac{\partial R_m}{\partial\Delta}$ by substituting (77) into (76) and solving as shown in

(78).

$$\frac{dR_m}{d\Delta} = -\frac{(\alpha\beta - \gamma C_m(\alpha) + R\gamma\Delta)}{16\alpha^6(h_{11}h_{22} - h_{12}^2)}(3\gamma C_m(\alpha)^2 + 2\alpha(\alpha\beta + 2R\gamma\Delta)C_m'(\alpha)$$
$$+ \alpha^2\gamma C_m'(\alpha)^2 + (\alpha\beta + 2R\gamma\Delta)(R\Delta - \alpha^2 C_m''(\alpha)) \tag{78}$$
$$+ C_m(\alpha)(-2\alpha\beta - 5R\gamma\Delta + \alpha\gamma(\alpha C_m''(\alpha) - \alpha C_m'(\alpha))))$$

Substituting $R\Delta = C_m(\alpha) - \alpha C_m'(\alpha)$ into (78) we have,

$$\frac{dR_m}{d\Delta} = \frac{(\alpha\beta - \gamma C_m(\alpha) + R\gamma\Delta)}{16\alpha^5(h_{11}h_{22} - h_{12}^2)}((\alpha C_m'(\alpha) - C_m(\alpha))(-\beta + \gamma C_m'(\alpha))$$

$$+ \alpha C_m''(\alpha)(\gamma C_m(\alpha) + \alpha(\beta - 2\gamma C_m'(\alpha)))) \tag{79}$$

The term $(\alpha\beta - \gamma C_m(\alpha) + R\gamma\Delta)$ is positive if the optimal demand is positive. The preceding

implies $\alpha\beta > \gamma(C_m(\alpha) - R\Delta)$. Considering $R\Delta = C_m(\alpha) - \alpha C_m'(\alpha)$ from (63), $\alpha\beta >$

$\gamma(C_m(\alpha) - C_m(\alpha) + \alpha C_m'(\alpha))$ i.e $\beta > \gamma C_m'(\alpha)$. Therefore $(\alpha C_m'(\alpha) - C_m(\alpha))(-\beta +$

$\gamma C_m'(\alpha)) > 0$. $(\gamma C_m(\alpha) + \alpha(\beta - 2\gamma C_m'(\alpha))) > 0$ since $\beta > \gamma C_m'(\alpha)$ and $C_m(\alpha) >$

$\alpha C_m'(\alpha)$. Therefore, we have $\frac{dR_m}{d\Delta} > 0$. The result implies that the optimal investment in

remanufacturability increases with cost savings for remanufacturing and is quite intuitive.

Substituting (77) into (76) and solving, we obtain an expression $\frac{d\alpha_m}{d\Delta}$ as shown in (80).

$$\frac{d\alpha_m}{d\Delta} = -\frac{(\alpha\beta + R\gamma\Delta - \gamma C_m(\alpha))}{16\alpha^5(h_{11}h_{22} - h_{12}^2)}(\Delta(\alpha\beta + R\gamma\Delta - \gamma C_m(\alpha)) + 4R\alpha^2 I''(R)) \tag{80}$$

From (80) since $\alpha\beta - \gamma C_m(\alpha) + R\gamma\Delta > 0$, $\frac{d\alpha_m}{d\Delta} < 0$. The result shows that the optimal product

life decreases with the costs savings per remanufacturing and illustrates the conflict between

a higher remanufacturability and a longer product life. To analyze how an increase in Δ

impacts the optimal demand for services, we can differentiate the expression for $\beta -$

$\gamma p(\hat{w}(R_m, \alpha_m), \alpha_m)$ from (74) with respect to Δ while noting that α_m and R_m should be

considered as functions of Δ as shown in (75).

$$\frac{d(\beta - \gamma p(\widehat{w}(R_m, \alpha_m), \alpha_m))}{d\Delta}$$

$$= \frac{\gamma \alpha_m (R_m \Delta + \gamma \frac{dR_m}{d\Delta}) + \frac{d\alpha_m}{d\Delta} \gamma(\alpha_m C_m'(\alpha_m) - C_m(\alpha_m) + R\Delta))}{4\alpha_m^2} \tag{81}$$

From (114), since $\alpha_m C_m'(\alpha_m) - C_m(\alpha_m) + R_m \Delta = 0$ and $\frac{dR_m}{d\Delta} > 0$ from (79), the optimal

demand for services $\beta - \gamma p(\widehat{w}(R_m, \alpha_m), \alpha_m)$ increases with Δ.

4.4 Comparison of MDSC with CCSC when product life is a variable

Given a constant product life, the traditional MDSC was inferior to CCSC with respect to

total supply chain profits, remanufacturability as well as the price to the customers.

Comparative analysis of MDSC with CCSC with variable product life can provide guidelines

on relative benefit of coordination when a higher remanufacturability or a longer product life

is preferred from the environmental standpoint.

4.4.1 The Centrally Coordinated Supply Chain

In this scenario, the manufacturer and the service provider are vertically integrated and the

decisions are made by a central planner with the objective of maximizing the total supply

chain profits. The relevant decision variables for this scenario are the price per unit service

p, the level of remanufacturability R and the product life α. The central planner's profit

maximization problem is formulated by (82).

$$\underset{p, \alpha, R}{Max} \pi_c = (\beta - \gamma p) - \frac{\beta - \gamma p}{\alpha}(C_m(\alpha) - R\Delta) - I(R) \tag{82}$$

Equations (83), (84) and (85) provide the first order conditions for π_c with respect to α, R

and p.

$$\frac{\partial \pi_c}{\partial \alpha} = -\left(\frac{\beta - \gamma p}{\alpha^2}\right)(\alpha C_m'(\alpha) - C_m(\alpha) + R\Delta) = 0 \qquad (83)$$

$$\frac{\partial \pi_c}{\partial R} = \Delta\left(\frac{\beta - \gamma p}{\alpha}\right) - I'(R) = 0 \qquad (84)$$

$$\frac{\partial \pi_c}{\partial p} = \beta - \gamma p + \frac{\gamma(C_m(\alpha) - R\Delta)}{\alpha} \qquad (85)$$

From (83), similar to MDSC given the optimal remanufacturability R_m, the optimal product

life α_m is independent of the price p and the demand parameters β and γ. This may be a result

of modeling the variable cost of manufacturing as a function of α. We will now derive

sufficient conditions for the existence of a quasi concavity of π_c. To show that π_c is quasi-

concave, it is sufficient to show that the hessian $D^2\pi_c(p, R, \alpha)$ as shown by (86) is negative

definite at any critical point. By critical point, we refer to any point where the first order

conditions are satisfied.

$$D^2\pi_c(p, R, \alpha) =$$
$$\begin{pmatrix} -2\gamma & -\frac{\gamma\Delta}{\alpha} & \frac{\gamma(R\Delta - C_m(\alpha) + \alpha C_m'(\alpha))}{\alpha^2} \\ -\frac{\gamma\Delta}{\alpha} & -I''(R) & -\frac{(\beta - \gamma p)\Delta}{\alpha^2} \\ \frac{\gamma(R\Delta - C_m(\alpha) + \alpha C_m'(\alpha))}{\alpha^2} & -\frac{(\beta - \gamma p)\Delta}{\alpha^2} & \frac{(\beta - \gamma p)(2R\Delta - 2C_m(\alpha) + 2\alpha C_m'(\alpha) - \alpha^2 C_m''(\alpha))}{\alpha^3} \end{pmatrix} \qquad (86)$$

Considering the hessian (86), the sufficient conditions for π_c to be concave at any critical

point are 1) $2\alpha^2 I''(R) > \gamma\Delta^2$ and 2) that $|D^2\pi_c(p, R, \alpha)|$ defined as the determinant of the

hessian $D^2\pi_c(p, R, \alpha)$ is less than zero. Simplifying the above conditions as in MDSC, a

sufficient condition for the quasiconcavity of π_c is that inequalities (87) and (88) are true in

the relevant range of R and α i. e. $0 \le R < 1$ and $\alpha_l \le \alpha$.

$$\Delta^2\beta - \gamma\Delta^2 C_m'(\alpha) - \alpha C_m''(\alpha)(2\alpha^2 I''(R) - \gamma\Delta^2)) < 0 \qquad (87)$$

$$2\alpha^2 I''(R) - \gamma\Delta^2 > 0 \qquad (88)$$

Note that the quasi-concavity conditions for CCSC (87) and (88) are relatively more restrictive than the corresponding conditions for MDSC. Also the conditions imply that π_c would be quasi-concave if $I''(R)$ is sufficiently large and maximum demand β is sufficiently small.

4.4.2 Comparison between MDSC and CCSC

PROPOSITION 2. *MDSC equilibrium has a lower remanufacturability, higher price per unit service and a greater product life compared to CCSC optimal solution.*

Previous articles have discussed the negative impact of double marginalization in decentralized supply chains such as reduced quality, higher prices and reduced profits [26, 27]. Conversely, Proposition 2 shows that the decentralized MDSC is superior to CCSC with respect to the product life. From an environmental standpoint given the higher price per unit service and the longer product life, it is possible that MDSC results in a lesser number of disposed products compared to CCSC. An intuitive interpretation for Proposition 2 is that the double marginalization in MDSC leads to lower demand for services which is an incentive for reducing the investment in remanufacturing. In turn, the reduced remanufacturability increases the average cost of manufacturing/ remanufacturing C_m -$R\Delta$ which is an incentive for a longer product life.

Proof: We will compare R_c and R_m, in the following steps. Step1: We show that suppose the manufacturer in MDSC sets $R = R_c$ instead of R_m, and then optimizes over product life and

wholesale price, the product life that maximizes his profit given $R = R_c$ is α_c. Step 2: We

show that given the manufacturer has set $R = R_c$ and is utilizing the optimal α and w given

that $R = R_c$, the manufacturer can increase his profit by reducing R. Given that π_m is quasi-

concave, step 2 implies that $R_m < R_c$.

Step 1: Since $\frac{\partial^2 \pi_c}{\partial p^2} = -2\gamma$, π_c is concave with respect to p and we can solve $\frac{\partial \pi_c}{\partial p} = 0$ to obtain

$\hat{p}(R, \alpha)$ the optimal price p for a given product life α and remanufacturability R as shown in

(89) (See Porteus [30] and Weng [36] for a similar approach).

$$\hat{p}(R, \alpha) = \frac{\alpha\beta + C_m(\alpha)\gamma - R\gamma\Delta}{2\alpha\gamma} \tag{89}$$

Substituting (89) in the profit (82), we can obtain an expression for $\pi_c(R, \alpha)$ defined as the

maximum CCSC profit that can be obtained by optimizing over price for a given R and α as

shown in (90).

$$\pi_c(R, \alpha) = \pi_c(\hat{p}(R, \alpha), R, \alpha) = \frac{(\alpha\beta - C_m(\alpha)\gamma + R\gamma\Delta)^2}{4\alpha^2\gamma} - I(R) \tag{90}$$

The first order conditions for maximizing $\pi_c(R, \alpha)$ are given by (91) and (92).

$$I'(R_c) = \frac{(\alpha_c \beta - C_m(\alpha_c)\gamma + R_c\gamma\Delta)}{2\alpha_c^2} \tag{91}$$

$$C_m(\alpha_c) = \alpha C'_m(\alpha_c) + R_c\Delta \tag{92}$$

Let us consider the manufacturer's profit $\pi_m(\alpha, R)$ as given by (67). Suppose that the

manufacturer utilizes the optimal remanufacturability of CCSC and sets $R=R_c$. We define

$\alpha_{|R=R_c}$ as the optimal product life that maximizes $\pi_m(\alpha, R)$ given that $R=R_c$. The first order condition satisfied by $\alpha_{|R=R_c}$ is given by (93).

$$\frac{\partial\pi_m(R,\alpha)|R=R_c}{\partial\alpha}$$

$$= \frac{(\alpha_{|R=R_c}\beta - C_m(\alpha_{|R=R_c})\gamma + R_c\gamma\Delta)(C_m(\alpha_{|R=R_c}) - \alpha C'_m(\alpha_{|R=R_c}) - R_c\Delta))}{4\alpha_{|R=R_c}{}^3} \quad (93)$$

$$= 0$$

Suppose that the term $(\alpha_{|R=R_c}\beta - C_m(\alpha_{|R=R_c})\gamma + R_c\gamma\Delta) = 0$ in (93), then considering (67), the optimal profit MDSC given $R= R_c$ and $\alpha = \alpha_{|R=R_c}$ is $-I(R_c)$. However, if the manufacturer sets $R=R_c$ and $\alpha=\alpha_c$, then since $(\alpha_c\beta - C_m(\alpha_c)\gamma + R_c\gamma\Delta) > 0$ from (67), the optimal profit is greater than $-I(R_c)$ contradicting that $\alpha_{|R=R_c}$ is the optimal product life in MDSC given $R = R_c$. Hence, $(\alpha_{|R=R_c}\beta - C_m(\alpha_{|R=R_c})\gamma + R_c\gamma\Delta) \neq 0$. Therefore, from (93), the optimal $\alpha_{|R=R_c}$ in MDSC satisfies (94).

$$C_m(\alpha_{|R=R_c}) - \alpha_{|R=R_c}C'_m(\alpha_{|R=R_c}) - R_c\Delta = 0 \quad (94)$$

Comparing (92) and (94), $\alpha_{|R=R_c} = \alpha_c$.

Step 2: the rate of change of $\pi_m(R, \alpha)$ with respect to R given that $R = R_c$ and $\alpha = \alpha_{|R=R_c}$ can be found by differentiating $\pi_m(\alpha, R)$ given by (67) with respect to α as shown in (95).

$$\frac{\partial\pi_m(R,\alpha)|R = R_c, \alpha_{|R=R_c}}{\partial R} = \frac{\Delta(\alpha_c)\beta - C_m(\alpha_c)\gamma + R_c\gamma\Delta}{4\alpha_c{}^2} - I'(R_c) \quad (95)$$

Substituting for $I'(R_c)$ in (91) from (95),

$$\frac{\partial \pi_m(R, \alpha)|R = R_c, \alpha_{|R=R_c}}{\partial R} = -\frac{\Delta(\alpha_c)\beta - C_m(\alpha_c)\gamma + R_c\gamma\Delta)}{4\alpha_c^2} < 0 \qquad (96)$$

Equation (96), implies that if the manufacturer sets $R=R_c$ and optimizes over α, then he can

gain profit by decreasing R. Given the manufacturer's profit is quasi-concave $R_m < R_c$.

To compare α_m and α_c, note that from (92) and (94) the α_m and α_c are related to the

remanufacturabilities R_m and R_c respectively via the same implicit function defined

by $C_m(\alpha) - \alpha C_m'(\alpha) - R\Delta = 0$. We can use the implicit function theorem to analyze how a

change in the remanufacturability affects the optimal product life. Let $C_m(\alpha) - \alpha C_m'(\alpha) -$

$R\Delta = G(\alpha, R) = 0$.

$$\frac{d\alpha}{dR} = \frac{-\frac{\partial G}{\partial R}}{\frac{\partial G}{\partial \alpha}} = -\frac{\Delta}{\alpha C_m''(\alpha)} < 0 \qquad (97)$$

Given that α_m and α_c are both related to R_m and R_c respectively via the same implicit function

defined by $G(\alpha, R) = C_m(\alpha) - \alpha C_m'(\alpha) - R\Delta = 0$, $\frac{d\alpha}{dR} < 0$ from (97) and $R_m < R_c$, we

have $\alpha_m > \alpha_c$. Hence, MDSC is superior with respect to CCSC with respect to the product

life although it results in lesser remanufacturing.

We will now compare the optimal prices in MDSC and CCSC. Note that the optimal supply

chain profit in CCSC $\pi_c(\alpha_c, R_c)$ is greater than the manufacturer's profit in MDSC $\pi_m(\alpha_m, R_m)$

by definition and considering that the CCSC has a unique optimal solution . From (90) and

(67),

$$\pi_c(R_c, \alpha_c) = \frac{(\alpha_c\beta - \gamma C_m(\alpha_c) + \gamma R_c\Delta)^2}{4\alpha_c{}^2\gamma} - I(R_c) > \pi_m(R_m, \alpha_m)$$

$$= \frac{(\alpha_m\beta - C_m(\alpha_m)\gamma + R_m\gamma\Delta)^2}{8\alpha_m{}^2\gamma} - I(R_m)$$

(98)

Since $R_c > R_m, I'(R_c) > I'(R_m)$. Therefore, we have

$$\frac{(\alpha_c\beta - \gamma C_m(\alpha_c) + \gamma R_c\Delta)^2}{4\alpha_c{}^2\gamma} > \frac{(\alpha_m\beta - C_m(\alpha_m)\gamma + R_m\gamma\Delta)^2}{8\alpha_m{}^2\gamma}$$

(99)

Taking the square root of (99),

$$\frac{(\alpha_c\beta - \gamma C_m(\alpha_c) + \gamma R_c\Delta)}{2\alpha_c} > \frac{(\alpha_m\beta - C_m(\alpha_m)\gamma + R_m\gamma\Delta)}{2\sqrt{2}\alpha_m}$$

(100)

From (64), the optimal demand in MDSC is given by

$$\beta - \gamma p_m = \frac{\alpha_m\beta - \gamma C_m(\alpha_m) + \gamma R_m\Delta}{4\alpha_m}$$

(101)

From (89),

$$\beta - \gamma p_c = \frac{\alpha_c\beta - \gamma C_m(\alpha_c) + \gamma R_c\Delta}{2\alpha_c}$$

(102)

Considering (100) and the expressions for the optimal demands (101) and (102) $\beta - \gamma p_c > \beta - \gamma p_m$ and $p_m > p_c$. In summary, given that the quasi-concavity conditions for π_c are met the CCSC will have a lower price per unit service, higher remanufacturability and lower product life as compared to MDSC.

4.5 Numerical Examples

We now provide a numerical example to illustrate the analytical insights and make further observations comparing MDSC and CCSC optimal solution. We assume that $I(R) = k(R^2)$ where k represents the investment required for achieving a remanufacturability of 1. Similar quadratic investment functions have been frequently used in the literature to model investments in setup, quality improvement and production costs savings[27, 33].

We assume $C_m(\alpha) = C_{m0} + s\alpha^2$ to relate the product life and the manufacturing cost. Here, C_{m0} represents manufacturing cost of long life components while $s\alpha^2$ represents the manufacturing cost of short life components that have the most impact on the product life.

We assumed hypothetical data for the various parameters. The parameter values utilized are as follows: $k = 1\,000\,000$, β=8 000 000, $\gamma = 2\,500\,000$, C_{m0} =12 000. We chose the parameters Δ and s for sensitivity analysis to observe how the equilibrium varies with conditions conducive for remanufacturing or a longer product life. We will first discuss the sensitivity with respect to Δ followed by the sensitivity for s.

We varied Δ from 2000 to 6775 while fixing $s = 0.00008$. Figures 8, 9 and 10 show the variation of R, p and α with cost savings per remanufactured product Δ. We observe that as analytically proved, $R_m < R_c$, $p_m > p_c$ and $\alpha_m > \alpha_c$. In addition, CCSC optimal solution is significantly more sensitive to Δ compared to MDSC equilibrium particularly at higher values of Δ. As a result, when Δ is high, the optimal CCSC solution has a substantially higher demand per services, higher remanufacturability and a lower product life compared to MDSC. An intuitive interpretation for this observation is as follows, in both MDSC and

CCSC, R is an increasing function of $\Delta(\beta - \gamma p)/\alpha$. Hence, if other variables are held constant, the sensitivity of R with Δ depends on the volume of products $(\beta - \gamma p)/\alpha$. Given that price and the product life are lower, the optimal volume of products is higher in CCSC. The higher volume of products over which the cost savings Δ can be achieved in CCSC explains the higher sensitivity to Δ to some extent.

A key objective of comparative analysis between MDSC and CCSC is to identify under what conditions is it more beneficial to coordinate MDSC. A measure of the benefit due to coordination is the percent gain in total supply chain profits ζ defined as $100(\pi_{c-}\pi_m)/\pi_m$. Since $\pi_{m<}\pi_c$, ζ>0. Figure 11 shows the variation of ζ with Δ. We observe that ζ is nonlinearly increasing with Δ suggesting that the benefit due to coordination is the highest when the conditions are favorable for remanufacturing. Note that the increase in ζ for MDSC with Δ was analytically shown by (48) when α was a parameter and $I(R) = kR^2$. A possible rationale for this observation is as follows. An increase in the cost savings Δ would be applicable to a greater volume of products in CCSC justifying higher investment in remanufacturability compared to MDSC. In turn, the higher remanufacturability leads to greater decrease in the product life and the price. Given the higher price per service and lower volume of products in MDSC due to double marginalization, these benefits of increasing the remanufacturability are realized to a lesser extent. Also note that the observed sensitivity of CCSC to Δ is higher than in the example in Chapter 3 where the product life was fixed at the optimal product life when $\Delta = 2000$. Perhaps, the option to reduce product life with Δ is why there is a greater increase in remanufacturability with increase in Δ.
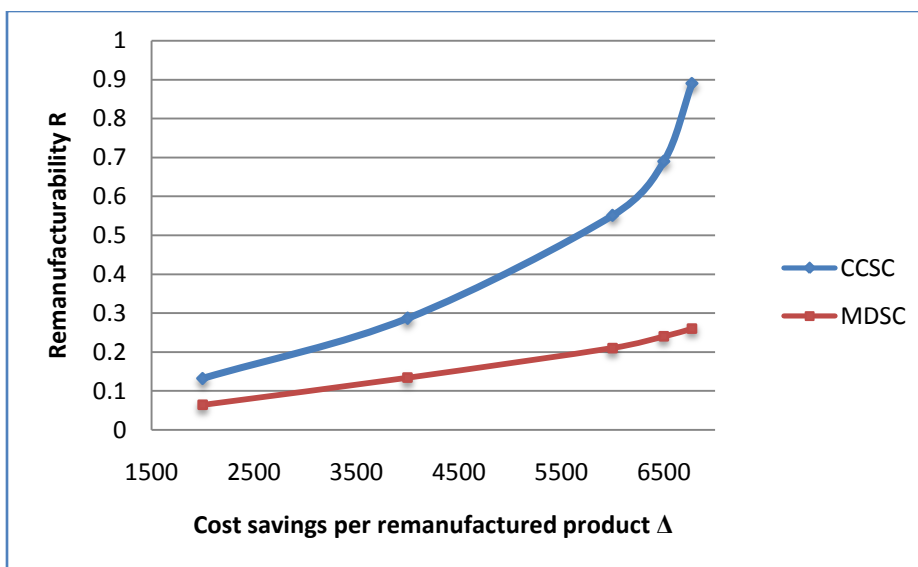
Figure 8 Variation of the Remanufacturability with Δ when α is a Variable
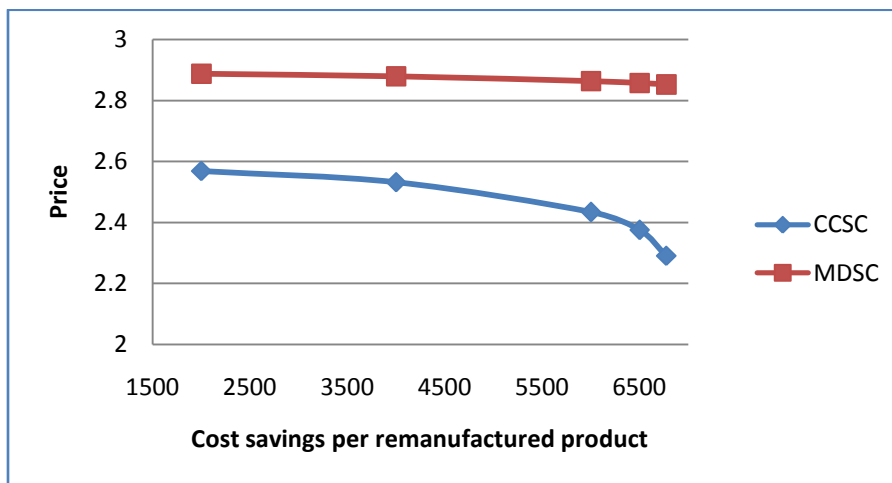

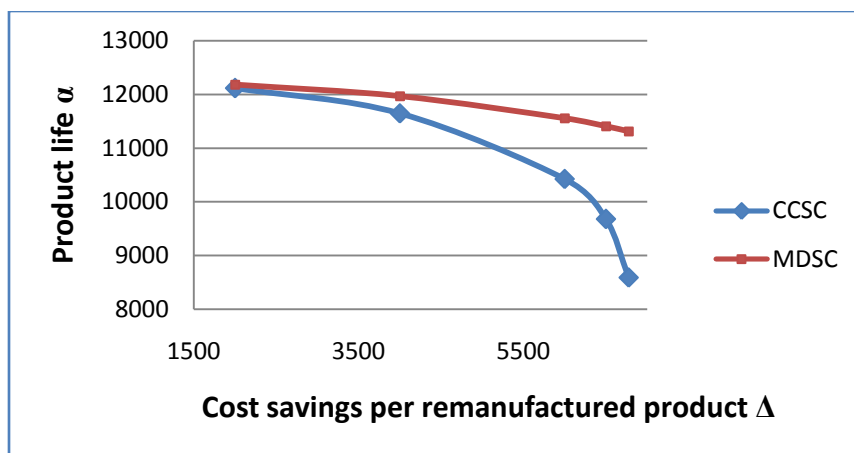**Figure 9 Variation of Price _p_ with Δ when α is Variable**

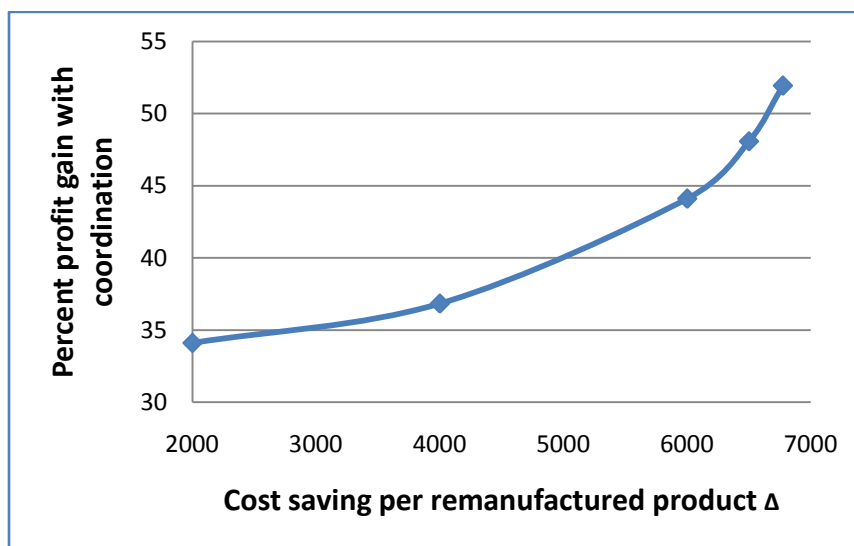**Figure 10 Variation of Product Life α with Δ**

Figure 11 Variation of Percent Profit Gain ζ with Δ

We will now observe how MDSC and CCSC solutions vary as it becomes expensive to have a longer product life. We varied *s* between 0.00004 and 0.00012 while fixing Δ at 6500\$/unit. Figure 12 shows that as expected, the product life in both MDSC and CCSC is decreasing as the parameter *s* of the manufacturing cost function $C_m(\alpha) = C_{m0} + s\alpha^2$ increases. Figure 12 Variation of Product Life α with Parameter *s*

Figure 14 shows the variation of the remanufacturabilities with increase in *s*. We observe that

CCSC remanufacturability is increasing with *s* when *s* is small and is decreasing with *s* when

s is large. As we discussed earlier, the optimal remanufacturability is a function of

$\Delta(\beta - \gamma p_m)/\alpha_m$, where $(\beta - \gamma p_m)/\alpha$ is the optimal demand for products. A possible

explanation for this observation is as follows, an increase in *s* causes two conflicting effects

impacting the optimal demand of products. First, an increase in *s* represent longer product

life related cost and can lead to decreased product life. Ceteris paribus, decreased product

life leads to increased volume of products. Conversely, since increase in *s* represents higher

manufacturing costs, it can lead to a decreased demand for services and products if α is

constant. The final sign of dR/ds depends on which of the two opposing effects: 1) the

decrease in demand of services and 2) decreased product life has a greater impact on the

optimal volume of products when all decision variables are jointly optimized. From

comparative statics analysis, we can derive the specific conditions under which the R_c

increases or decreases with product life as shown in (103) and (104). From (104) we can

observe that R_c decreases with *s* if the optimal demand for services is sufficiently small.

$$\frac{dR_c}{ds} > 0 \; if \; (\alpha\beta - C_{m0}\gamma - \gamma s\alpha^2 + R\gamma\Delta) > n\gamma(C_{m0} - R\Delta) \qquad (103)$$

$$\frac{dR_c}{ds} \leq \; if \; (\alpha\beta - C_{m0}\gamma - \gamma s\alpha^2 + R\gamma\Delta) \leq n\gamma(C_{m0} - R\Delta) \qquad (104)$$
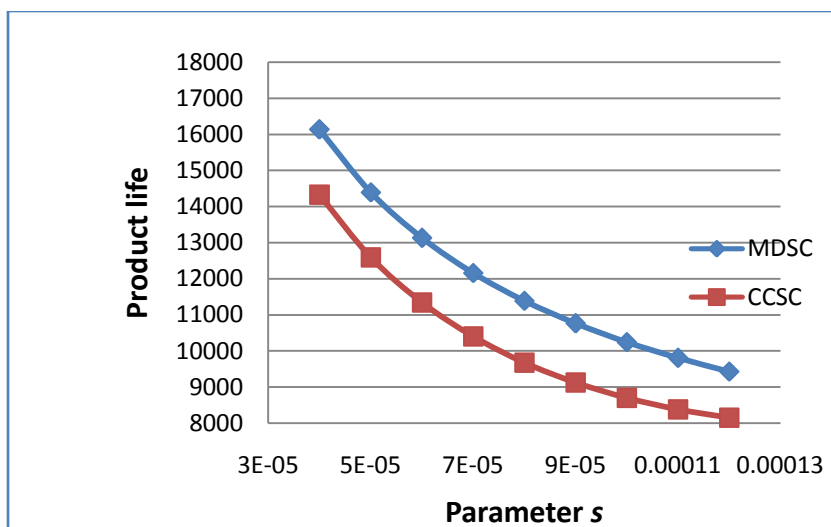
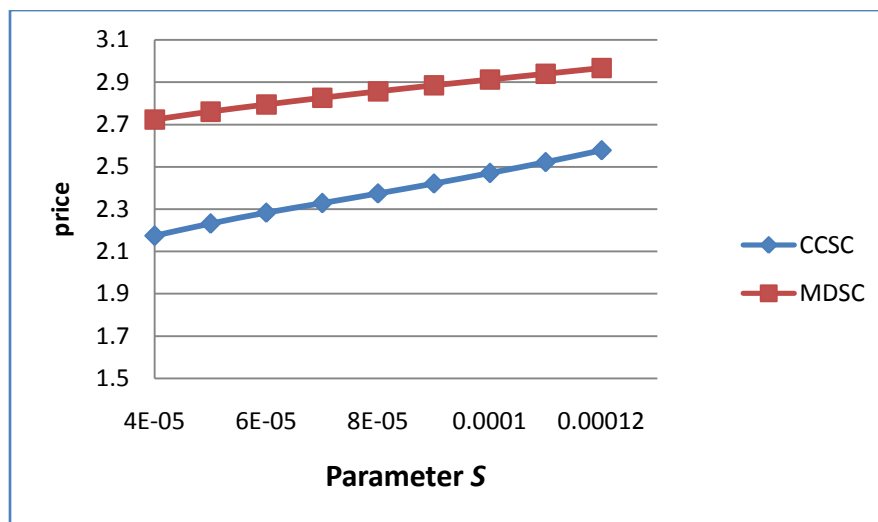**Figure 12 Variation of Product Life α with Parameter *s***

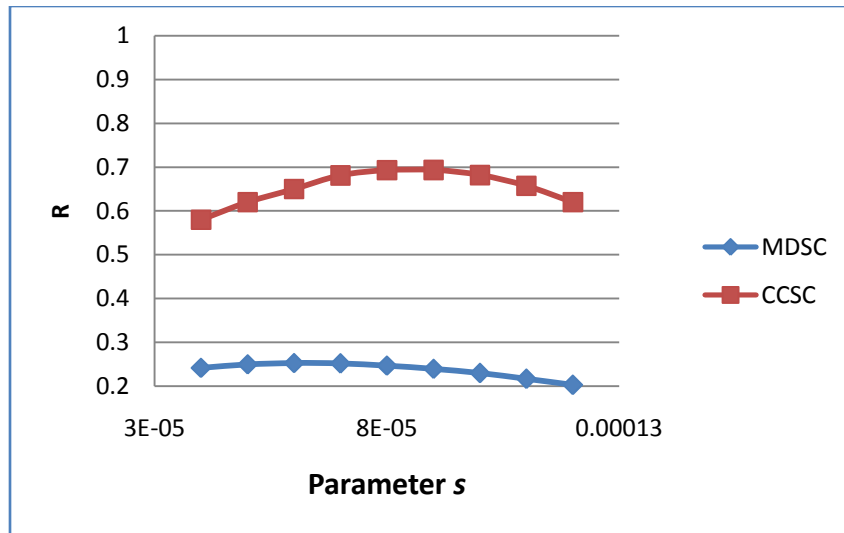**Figure 13 Variation of Price *p* with Parameter *s***

Figure 14 Variation of the Remanufacturability *R* with Parameter *s*

Figure 15 shows the variation of the percent profit gain ζ with increase in *s*. We can observe

that ζ is almost linearly increasing with *s*. An intuitive interpretation of the above result is

that given the significantly higher remanufacturability and a lower product life, CCSC is

much less susceptible to increase in product life associated manufacturing cost compared to

MDSC. Overall in this example we observed that the difference in the solutions of MDSC

and CCSC are the highest when the cost savings per remanufactured product Δ is high. In

addition, we observed that it is most beneficial to coordinate when Δ is high or when it is
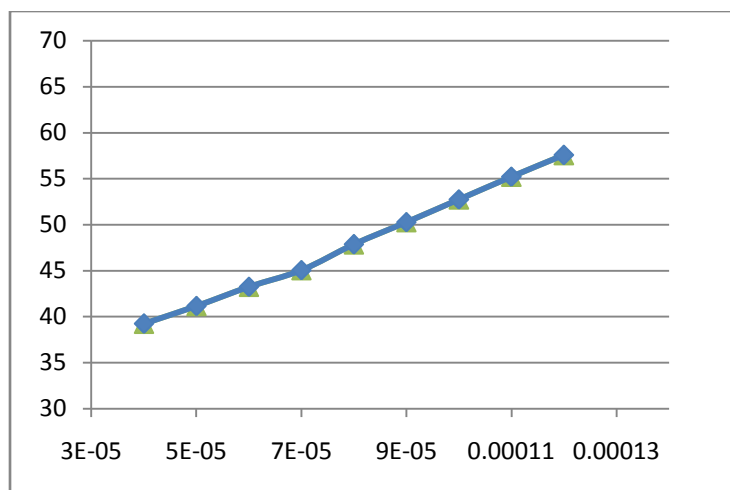
expensive to have a longer product life (*s* is high).

Figure 15 Variation of the Percent Profit Gain via Coordination ζ with the Parameter *s*

5. Impact of Environmental Legislation

In this chapter, we examine how a specific type of environmental legislation that is aimed at reducing the waste impacts MDSC and CCSC. Specifically, we consider the scenario where the government imposes an environmental fee σ (dollars/product) for each product that is disposed into the waste stream. The fee σ can be utilized for various purposes such as recycling, environmental disposal and consumer education. This fee structure is influenced by the recent e-waste legislation in Maine where the producers pay for recycling of their products[20].

5. 1 Analysis with Product life as a Parameter

Recall that the number of disposed products is $(1 - R)(\beta - \gamma p)/\alpha$. Given an environmental fee of σ, the total cost due to the environmental fee is $\sigma(1 - R)(\beta - \gamma p)/\alpha$. Considering σ, the manufacturer's profit maximization problem (5) is modified as shown in (105).

$$\underset{w, R}{Max} \; \pi_m^m = \frac{(\alpha\beta - w\gamma)}{2\alpha^2}(w - C_m - \sigma + R(\Delta + \sigma)) - I(R) \tag{105}$$

In this case, the manufactures objective would be concave and we would have a unique Stackelberg equilibrium if

$$4\alpha^2 I''(R^*) > \gamma(\Delta + \sigma)^2 \tag{106}$$

PROPOSITION 3. *When product life is constant, the equilibrium remanufacturability in MDSC decreases with the environmental fee σ if the marginal demand γ is sufficiently high. When γ is sufficiently low, the remanufacturability in MDSC increases with σ.*

One of the objectives of legislation imposing the environmental fee σ is to encourage product reuse. However, Proposition 3 shows when the marginal demand is high, the environmental fee leads to the undesirable outcome of reduced investment in remanufacturing. A possible explanation for this result is that when γ is relatively high, the optimal demand for services and products is relatively low implying a lesser incentive to invest in remanufacturability. Under the circumstances, given an increase in σ, it may be more economical to reduce the number of disposed products $(1 - R)(\beta - \gamma p)/\alpha$ via reducing price as opposed to increasing the remanufacturability.

Proof: Equation(14) shows $D^2\pi_m^m$, defined as the hessian matrix for π_m^m with respect to *w* and *R*.

$$D^2\pi_m^m = \begin{pmatrix} -\dfrac{\gamma}{\alpha^2} & -\dfrac{\gamma(\Delta + \sigma)}{2\alpha^2} \\ -\dfrac{\gamma(\Delta + \sigma)}{2\alpha^2} & -I^{''}(R) \end{pmatrix} \tag{107}$$

The implicit function theorem is applicable if we have an interior equilibrium i.e. the first order conditions are met and $|D^2\pi_m^m| \neq 0$. Here, $|D^2\pi_m^m|$ denotes the determinant of the hessian $D^2\pi_{m..}^m$ From the implicit function theorem we have,

$$
\begin{pmatrix} \dfrac{\partial w_m}{\partial \sigma} \\[2ex] \dfrac{\partial R_m}{\partial \sigma} \end{pmatrix} = \begin{pmatrix} \dfrac{\partial^2 \pi_m^m}{\partial w^2} & \dfrac{\partial^2 \pi_m^m}{\partial w \partial R} \\[2ex] \dfrac{\partial^2 \pi_m^m}{\partial R \partial w} & \dfrac{\partial^2 \pi_m^m}{\partial R^2} \end{pmatrix}^{-1} \begin{pmatrix} -\dfrac{\partial^2 \pi_m^m}{\partial w \partial \sigma} \\[2ex] -\dfrac{\partial^2 \pi_m^m}{\partial R \partial \sigma} \end{pmatrix} \tag{108}
$$

Where,

$$
\begin{pmatrix} \dfrac{\partial^2 \pi_m^m}{\partial w \partial \sigma} \\[2ex] \dfrac{\partial^2 \pi_m^m}{\partial R \partial \sigma} \end{pmatrix} = \begin{pmatrix} \dfrac{(R-1)}{2\alpha^2} \\[2ex] \dfrac{\alpha\beta - w\gamma}{2\alpha^2} \end{pmatrix} \tag{109}
$$

Substituting (107) and (109) in (108) and solving, we obtain

$$
\frac{dR_m}{d\sigma} = \frac{2\alpha\beta - 2w\gamma - \gamma(1-R)(\Delta + \sigma)}{4\alpha^2 I''(R_m) - \gamma(\sigma + \Delta)^2} \tag{110}
$$

Solving $\dfrac{\partial \pi_m^m}{\partial w} = 0$, we obtain

$$
w_m = \frac{\alpha\beta + \gamma C_m - R_m \gamma \Delta - \gamma\sigma(R_m - 1)}{2\gamma} \tag{111}
$$

Substituting (111) into (110) and simplifying,

$$
\frac{dR_m}{d\sigma} = \frac{(\alpha\beta - C_m\gamma + R_m\gamma\Delta - \gamma\sigma(1-R_m)) - \gamma(1-R_m)(\Delta + \sigma)}{4\alpha^2 I''(R_m) - \gamma(\sigma + \Delta)^2} \tag{112}
$$

From (112), $\dfrac{dR_m}{d\sigma} < 0$ if γ is sufficiently high since R_m decreases with γ from (17). $\dfrac{dR_m}{d\sigma} \geq 0$

otherwise. Substituting (107) and (109) in (108) and solving, we obtain

$$
\frac{dw_m}{d\sigma} = \frac{2(1-R)\alpha^2 I''(R_m) - (\alpha\beta - w\gamma)(\sigma + \Delta)}{4\alpha^2 I''(R_m) - \gamma(\Delta + \sigma)^2} \tag{113}
$$

Substituting the first order condition (111) into (113) we obtain,

$$\frac{dw_m}{d\sigma} = \frac{4(1-R)\alpha^2 I''(R_m) - (\alpha\beta - C_m\gamma + R_m\gamma\Delta - \gamma\sigma(1-R_m))(\sigma + \Delta)}{8\alpha^2 I''(R_m) - 2\gamma(\Delta + \sigma)^2} \quad (114)$$

From (114), $\frac{dw_m}{d\sigma} > 0$ if γ is sufficiently high otherwise $\frac{dw_m}{d\sigma} < 0$. An intuitive interpretation

for this result is that when γ is sufficiently low, the demand for services and products is high

and there is a greater incentive for investment in remanufacturing. Under the circumstances

reducing the wholesale price to support remanufacturing may be beneficial. Note that

considering the service provider's best response function (3) and given a constant product

life $\frac{dp_m}{d\sigma} > 0$ if $\frac{dw_m}{d\sigma} > 0$ and $\frac{dp_m}{d\sigma} \leq 0$ if $\frac{dw_m}{d\sigma} \leq 0$.

We will now illustrate the above results with a numerical example. The parameters for this

example are similar to those for section 4. The specific values are k = 1 000 000, Cm =

23737, γ = 2 500 000, α = 12113 and Δ = 5000 . We then varied the environmental fee σ

between 0 and 800 for maximum demand parameter β values 5 000 000 and 8 000 000.

 shows the variation of R and p with σ when β is 5 000 000. We observe that an increase in

σ is resulting in a decrease in R and an increase in p. A possible rationale for this

observation is that, when α is low, the optimal demand for services is low. Under the

circumstances, increasing p is more economical as a strategy to reduce the number of

disposed products $(1 - R)(\beta - \gamma p)/\alpha$ compared to increasing the R.
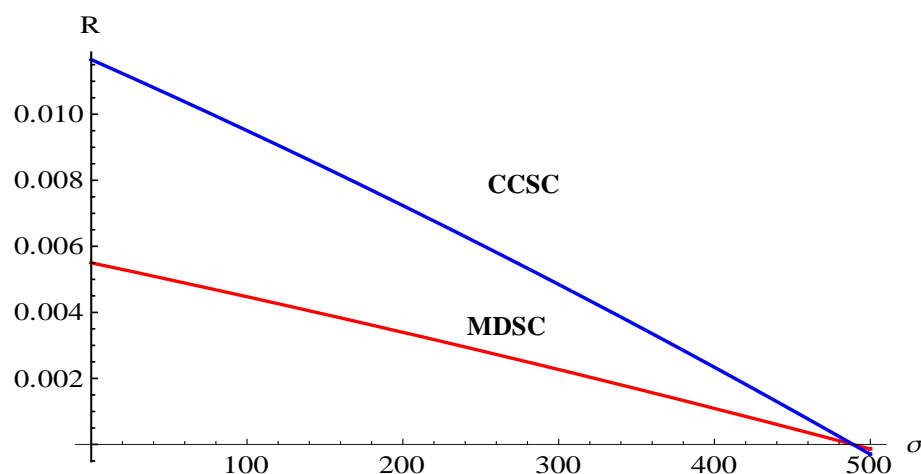
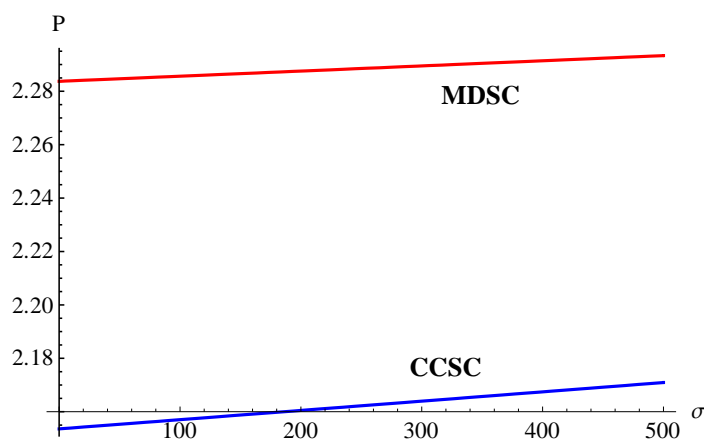**Figure 16 Variation of *R* with *σ* under MDSC and CCSC with β = 5 000 000**

Figure 17 Variation of *p* with *σ* under MDSC and CCSC with *β* = 5 000 000

Figure 18 shows the variation of the number of disposed products $(1 - R)(\beta - \gamma p)/\alpha$ with

σ. In this case the observed decrease in the number of products disposed is caused by an

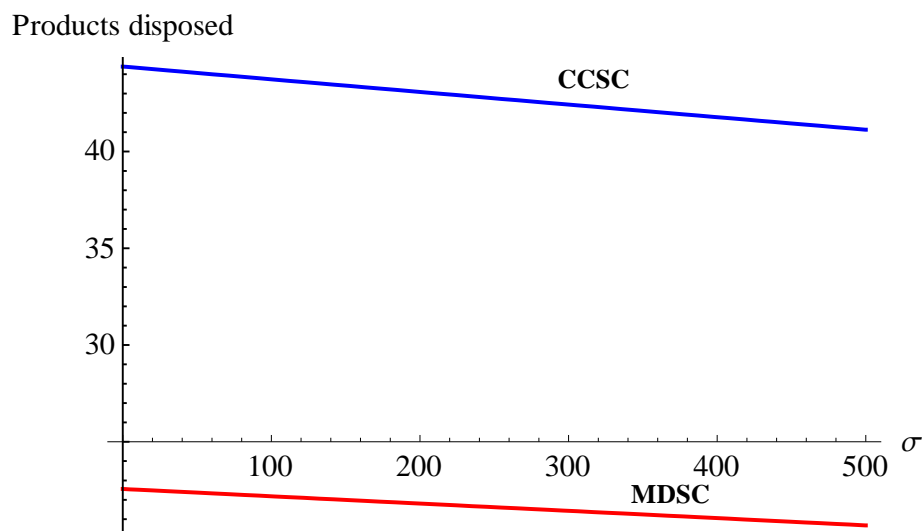increase in price although the *R* decreased.

Figure 18 Variation of Products Disposed with σ under MDSC and CCSC with β= 5 000 000

Figure 20 and Figure 20 shows the variation of R and p with σ when β is 8 000 000. We

observe that an increase in σ is resulting in increased R as well as p. The variation of the

number of number of disposed products $(1 - R)(\beta - \gamma p)/\alpha$ with σ is shown in Figure 21. In

this case, the price p is relatively incentive σ and the decrease in the number of products
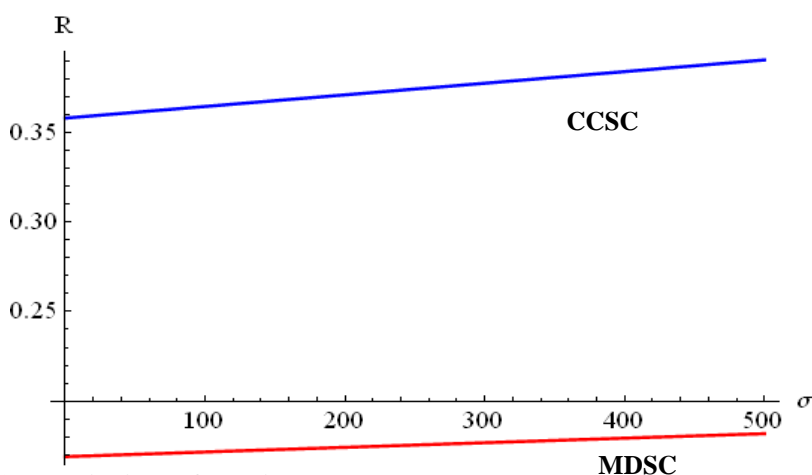
disposed is primarily due to the increase in R.

Figure 19 Variation of R with σ under MDSC and CCSC with β = 8 000 000
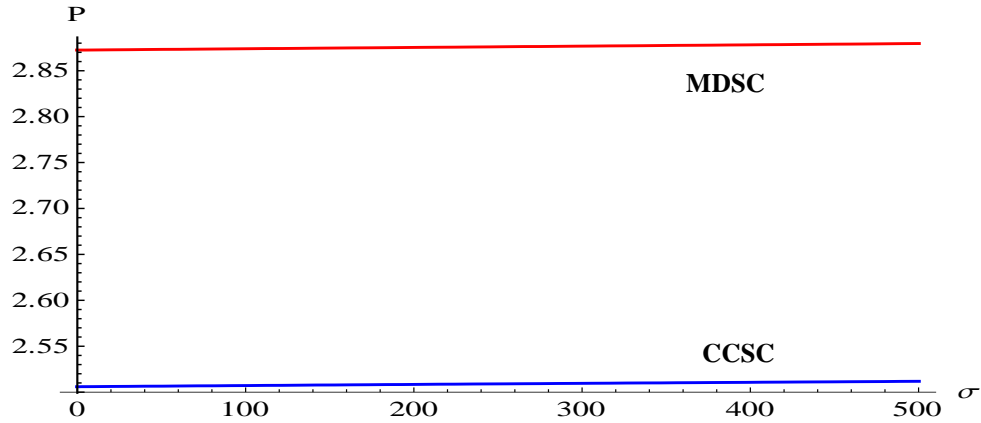
Figure 20 Variation of *p* with *σ* under MDSC and CCSC with β = 8 000 000
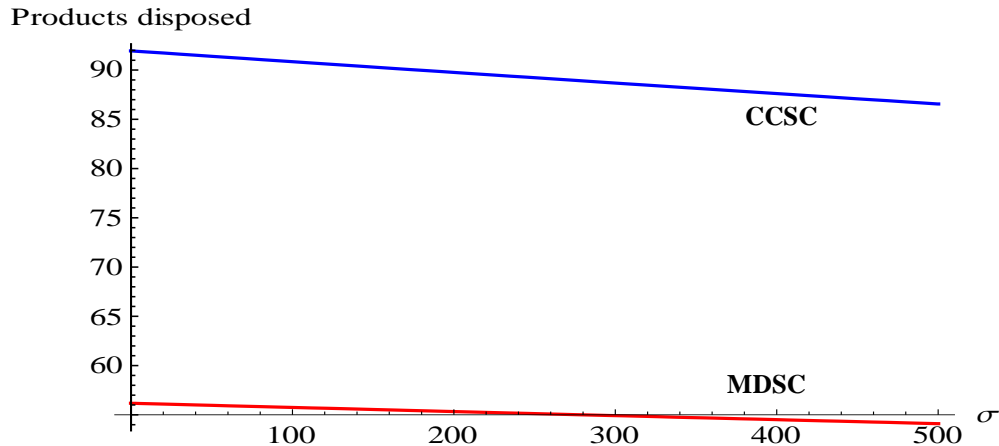
Figure 21 Variation of Products Disposed with *σ* under MDSC and CCSC with β = 8 000 000

5.2 Numerical Example with Product Life as a Variable

In this section, we will provide a couple of illustrative numerical examples to observe the impact of the environmental fee when product life is a variable. Given the environmental fee σ, the manufacturer's profit maximization problem considering the service provider's best response function is provided by (115).

$$\underset{w, R, \alpha}{Max} \ \pi_m = \frac{(\beta - \gamma p(w, \alpha))}{\alpha} (w - C_m(\alpha) + \sigma + R(\Delta + \sigma)) - I(R) \qquad (115)$$

The sufficient conditions for the quasiconcavity of the manufacturer's profit π_m^m are provided by Inequalities (116) and (106).

$$(\Delta + \sigma)^2 \beta - \gamma(\Delta + \sigma)^2 C_m'(\alpha) - \alpha C_m''(\alpha)(4\alpha^2 I''(R) - \gamma(\Delta + \sigma)^2))$$
$$< 0 \tag{116}$$

We assume that $I(R) = k(R^2)$ and $C_m(\alpha) = C_{m0} + s\alpha^2$. The parameters for this example are based on the hypothetical data assumed in the example 1 of Section 6.4. The specific parameter values are: $k = 1000\,000$, β=800 0000, $\gamma = 2\,500\,000$, C_{m0}=12 000 and $\alpha = 12113$. Figure 22, Figure 23 and Figure 24 show the variation of R, α, and p with the environmental fee σ. We can observe that in CCSC, R is increasing with σ while α and p are decreasing with σ. In this case, reducing the number of disposed products $(1 - R)(\beta - \gamma p)/\alpha$ via increasing the remanufacturability is more economical than the alternative strategies of increasing the price or the product life. Furthermore, the product life and the price are decreased such that the cost savings due to increased R are applicable to a greater volume of products$(\beta - \gamma p)/\alpha$.

In MDSC, the R is marginally increasing with σ while α and p are increasing with σ. A possible rationale for the observation is that in MDSC, the demand for products is low due to the higher price and product life. Under the circumstances increasing the product life is more appropriate as a strategy to reduce waste than in CCSC where the demand for products is higher. We note the preceding difference in the variation of α with the environmental fee σ in CCSC and MDSC may be a result of the specific parameters assumed in this example.
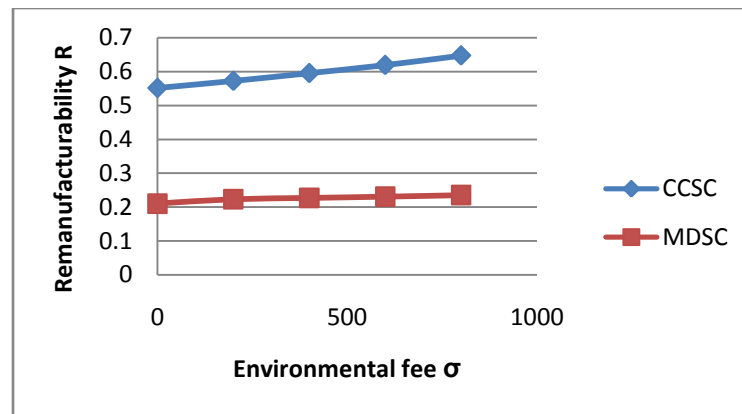
Figure 22 Variation of *R* with *σ* under MDSC and CCSC when α is Variable
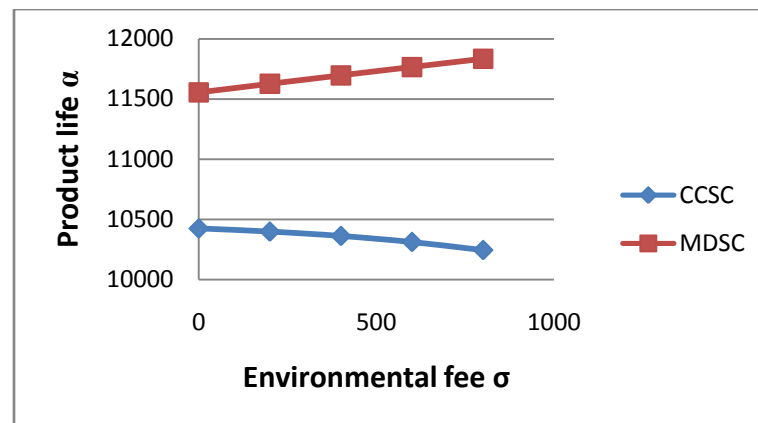
Figure 23 Variation of α with *σ* under MDSC and CCSC
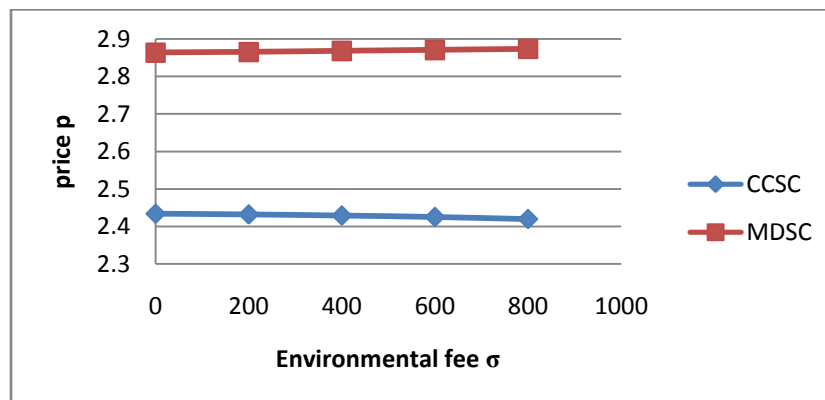
Figure 24 Variation of *p* with *σ* under MDSC and CCSC when α is Variable

Figure 25 shows the variation of the number of products disposed in MDSC and CCSC with σ. We can observe that CCSC has higher number of disposed products compared to MDSC although it has a higher R because of the lower α and $p.$ Also, there is a greater decrease in the number of disposed products with σ in CCSC.
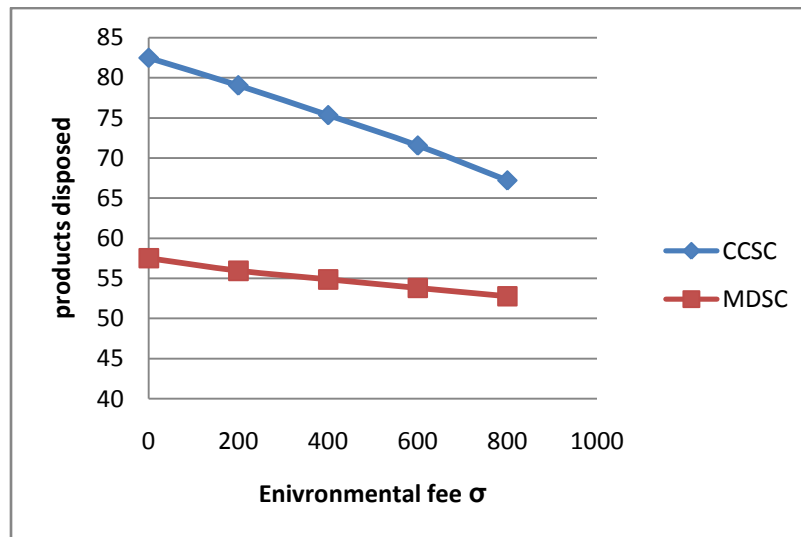
Figure 25 Variation of the Number of Products Disposed with σ under CCSC and MDSC when α = 12113

6. The Service Provider Driven Supply Chain (SPDSC)

In recent years, the service provider driven supply chain (SPDSC) has gained relevance as service providers are becoming increasingly powerful by organization of multiple service centers into service provider chains. For example, large service provider chains such as Fedex Kinkos and Staples are likely to have a considerably high supply chain power. Although retailers and service providers are becoming more dominant relative to the manufacturer, only a small minority of papers in the supply chain literature have considered a dominant downstream supply chain member [12, 13, 21].

In this study, we utilize the Stackelberg game with the Service provider as the leader and the manufacturer as the follower to model this scenario. However, unlike the case of MDSC, we cannot obtain reasonable equilibrium if we represent the strategies of the manufacturer and the service provider in terms of wholesale price *w* and price per unit service *p* alone. The difficulty in doing so is that, if the service provider first decides the price *p* as the Stackelberg leader, the manufacturer as the follower can charge a large enough wholesale price to make the service provider's margin and thus profit zero. To overcome this issue, vertical supply chain articles using Stackelberg game model with the retailer as the leader have utilized margins instead of the prices as the player's strategies [12, 26]. These margins can either be absolute margins in dollars or a percentage margin. Given that both absolute and percentage margins are equally possible in practice, we chose absolute margins as described in [12, 13, 26].

Kadiyali *et al* [37] tested the margins predicted by game theoretic models of vertical supply chains such as the cournot model, the manufacturer leader Stackelberg model and the service

provider leader Stackelberg model. The theoretical models tested were based upon Choi's model [29] of two competing manufacturers and a single retailer while the empirical data used was for two products orange juice and tuna. The article found that the retailer Stackelberg model with absolute margins fit the data better than Cournot game or the manufacturer Stackelberg game. Additionally, the observed percentage of the retailer profits (around 60%) was much higher than that predicted by the manufacturer Stackelberg model. These results further support use of margins instead of prices as the player's strategy when a downstream supply chain member is the leader.

A question regarding the practical relevance of SPDSC is, whether the service provider's pricing strategies can influence the manufacturer's technology investment as implied by SPDSC. While formal empirical research in this area is scarce, anecdotal evidence from trade journals suggests that in the retailer-manufacturer supply chain, retailer's pricing could impact the manufacturer's technologies. A news article comments "Wal-Mart is pushing the RFID technology on the idea it will increase efficiency and eventually save everyone money -- manufacturers as well as Wal-Mart. The suppliers have had to bear the cost of buying hardware -- readers, transponders, antennas -- and computer software to track and analyze the data" [7]. In a similar fashion, perhaps powerful service providers may influence the manufacturer's investments in cost saving technology.

We will now discuss the solution of SPDSC model where we utilize the service provider leader Stackelberg game as an initial benchmark for the supply chain behavior when the service provider dominates the pricing game.

6.1 Stackelberg Game Formulation of SPDSC

In this game, the service provider (leader) first decides her absolute margin per unit service $g= p-w/\alpha$. The manufacturer being the follower accepts the service provider's margin and decides the wholesale price w and the remanufacturability R that maximize his profits. In this case, determining the wholesale price is equivalent to deciding the price per unit service given $p= g+w/\alpha$.

As in the literature, to determine the Stackelberg equilibrium, we first characterize the manufacturer's reaction function i.e., the wholesale price $w(g)$ and remanufacturability $R(g)$ that maximize his profits given the service providers margin g. We then determine the optimal g_{sp} that maximizes the service provider's profit while considering $w(g)$. Note given that the reaction function $w(g)$, the reaction function $R(g)$ is irrelevant to the service provider's profit and hence she ignores it in her profit maximization. The description of this SC is summarized in Figure 26.

.In what follows, we will derive the equilibrium conditions which will form the basis for comparing the different SC. In addition, we will derive the conditions for the concavity of the manufacturer's and service provider's objectives which are sufficient conditions for existence of the reaction function and the uniqueness of equilibrium respectively[31].
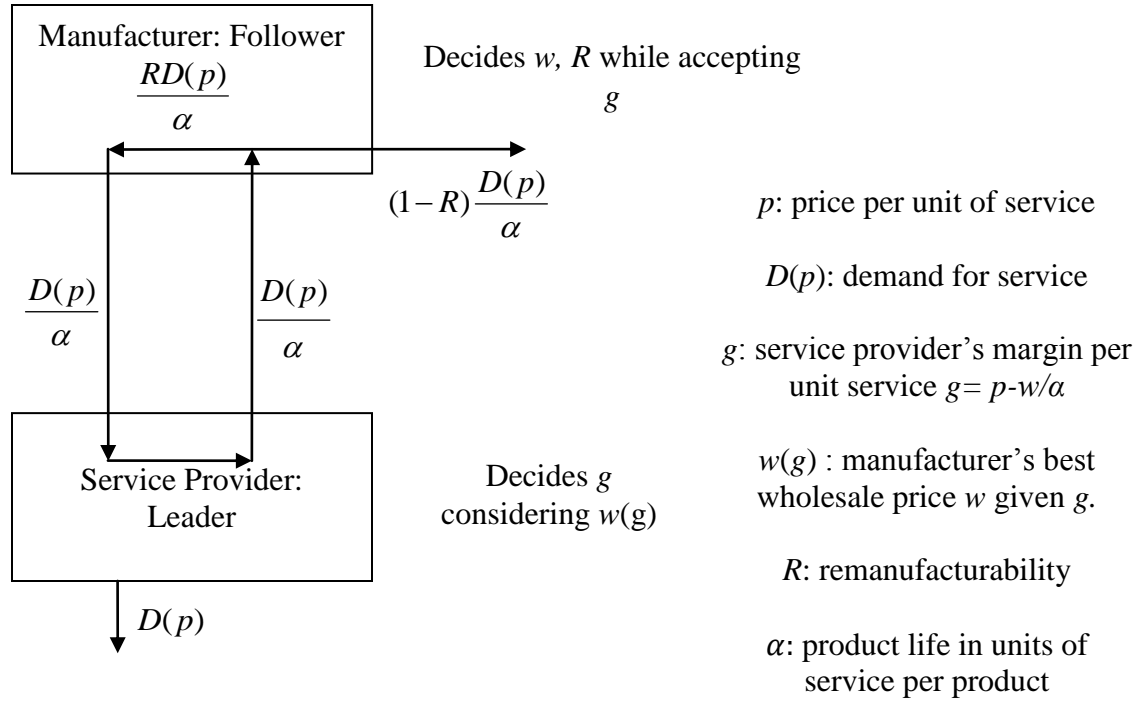
Figure 26 Service Provider Driven Supply Chain (SPDSC)

The profit maximization problem for the manufacturer is given by (117). Here

$(\beta - \gamma(g + \frac{w}{\alpha}))$ is the demand for services given the service provider's margin g and

wholesale price w.

$$\underset{w,R}{Max} \quad \pi_{m}^{sp} = \frac{(\beta - \gamma(g + \frac{w}{\alpha}))}{\alpha}(w - C_m + R\Delta) - I(R) \tag{117}$$

The first derivatives of the manufacturer's profits are given by (118) and (119). The

optimality conditions for the manufacturer are found by equating the first derivatives to zero.

$$\frac{\partial \pi_m^{sp}}{\partial w} = \frac{\alpha(\beta - \gamma g) + \gamma(C_m - 2w - R\Delta)}{\alpha^2} \tag{118}$$

$$\frac{\partial \pi_m^{sp}}{\partial R} = \frac{(\beta - \gamma(g + \frac{w}{\alpha}))\Delta}{\alpha} - I'(R) \tag{119}$$

The Hessian matrix of the manufacturer's profit is

$$\begin{pmatrix} -\dfrac{2\gamma}{\alpha^2} & -\dfrac{\gamma\Delta}{\alpha^2} \\ -\dfrac{\gamma\Delta}{\alpha^2} & -I''(R_{sp}) \end{pmatrix} \tag{120}$$

From Hessian (120), the manufacturer's objective is strictly concave if

$$I''(R) > \frac{\gamma\Delta^2}{4\alpha^2} \tag{121}$$

Considering, (121), service providers profit is concave if the curvature $I''(R)$ of the

investment function $I(R)$ is relatively large. A non concave objective function may result in a

reaction function that is not differentiable or in worst case may not exist (several points have

the same profit). The preceding makes the leaders optimization very complicated. Hence,

throughout this paper we will only consider the case when inequality (121) is true. We will

now characterize the manufacturer's reaction functions that are necessary for the service

provider's optimization.

Given that Inequality (121) is satisfied and assuming there exist w and R such that the first

derivatives given by (118) and (119) are zero (i.e. assuming that we have an interior

solution) $\dfrac{\partial w(g)}{\partial g}$ and $\dfrac{\partial R(g)}{\partial g}$ are given by the unique solution of (122) according to the

implicit function theorem (see [31, 32] for the statement of implicit function theorem). The

resulting expressions for $\dfrac{\partial w(g)}{\partial g}$ and $\dfrac{\partial R(g)}{\partial g}$ are given by (125) and (126).

$$
\begin{pmatrix}
\dfrac{\partial^2 \pi_m^{sp}}{\partial w^2} & \dfrac{\partial^2 \pi_m^{sp}}{\partial R \partial w} \\[3mm]
\dfrac{\partial^2 \pi_{sp}^{m}}{\partial R \partial w} & \dfrac{\partial^2 \pi_{sp}^{m}}{\partial R^2}
\end{pmatrix}
\begin{pmatrix}
\dfrac{\partial w}{\partial g} \\[3mm]
\dfrac{\partial R}{\partial g}
\end{pmatrix}
= -
\begin{pmatrix}
\dfrac{\partial \pi_m^{sp}}{\partial w \partial g} \\[3mm]
\dfrac{\partial \pi_m^{sp}}{\partial R \partial g}
\end{pmatrix}
\tag{122}
$$

$$
\begin{pmatrix}
-\dfrac{2\gamma}{\alpha^2} & -\dfrac{\gamma\Delta}{\alpha^2} \\[3mm]
-\dfrac{\gamma\Delta}{\alpha^2} & -I''(R_{sp})
\end{pmatrix}
\begin{pmatrix}
\dfrac{\partial w_{sp}}{\partial g} \\[3mm]
\dfrac{\partial R_{sp}}{\partial g}
\end{pmatrix}
=
\begin{pmatrix}
\dfrac{\gamma}{\alpha} \\[3mm]
\dfrac{\gamma\Delta}{\alpha}
\end{pmatrix}
\tag{123}
$$

$$
\begin{pmatrix}
\dfrac{\partial w_{sp}}{\partial g} \\[3mm]
\dfrac{\partial R_{sp}}{\partial g}
\end{pmatrix}
=
\begin{pmatrix}
\dfrac{-I''(R)}{\dfrac{2\gamma I''(R_{sp})}{\alpha^2} - \left(\dfrac{\gamma\Delta}{\alpha^2}\right)^2} & \dfrac{\dfrac{\gamma\Delta}{\alpha^2}}{\dfrac{2\gamma I''(R_{sp})}{\alpha^2} - \left(\dfrac{\gamma\Delta}{\alpha^2}\right)^2} \\[6mm]
\dfrac{\dfrac{\gamma\Delta}{\alpha^2}}{\dfrac{2\gamma I''(R_{sp})}{\alpha^2} - \left(\dfrac{\gamma\Delta}{\alpha^2}\right)^2} & \dfrac{\dfrac{-2\gamma}{\alpha^2}}{\dfrac{2\gamma I''(R_{sp})}{\alpha^2} - \left(\dfrac{\gamma\Delta}{\alpha^2}\right)^2}
\end{pmatrix}
\begin{pmatrix}
\dfrac{\gamma}{\alpha} \\[3mm]
\dfrac{\gamma\Delta}{\alpha}
\end{pmatrix}
\tag{124}
$$

Solving (124),

$$
\frac{\partial w_{sp}(g)}{\partial g} = \frac{\alpha(\gamma\Delta^2 - \alpha^2 I''(R_{sp}))}{-\gamma\Delta^2 + 2\alpha^2 I''(R_{sp})}
\tag{125}
$$

$$\frac{\partial R_{sp}(g)}{\partial g} = \frac{\alpha\gamma\Delta}{\gamma\Delta^2 - 2\alpha^2 I''(R_{sp})} \tag{126}$$

From (126), we can observe that the manufacturer decreases the remanufacturability with increase in service provider's margin g. The manufacturer's reaction function with respect to w is more complicated. If $I''(R_{sp})$ is sufficiently high, then he decreases w with increase in g while he would increase w with increase in g if $I''(R_{sp})$ is sufficiently low. Note that for the manufacturer, deciding the wholesale price w is equivalent to choosing the price per service p as the service provider has already specified her margin g. Specifically, we have that

$$p = g + \frac{w}{\alpha} \tag{127}$$

$$\frac{\partial p}{\partial g} = 1 + \frac{(\gamma\Delta^2 - \alpha^2 I''(R_{sp}))}{-\gamma\Delta^2 + 2\alpha^2 I''(R_{sp})} \tag{128}$$

$$\frac{\partial p}{\partial g} = \frac{\alpha^2 I''(R_{sp}))}{-\gamma\Delta^2 + 2\alpha^2 I''(R_{sp})} \tag{129}$$

From (129) we can infer that an increase in the service provider's margin g would lead to an increase in the price to the customers as far as manufacturer's reaction is concerned. The service provider being the leader chooses the optimal margin g that maximizes her profit as given by.

$$\underset{g}{Max} \quad \pi_{sp}^{sp} = (\beta - \gamma(\frac{w(g)}{\alpha} + g))g \tag{130}$$

Equations (131) and (132) provide the first derivatives of the service provider's objective.

$$\frac{\partial \pi_{sp}^{sp}}{\partial g} = \beta - \gamma(\frac{w(g)}{\alpha} + g) - \gamma g(\frac{\partial w(g)}{\alpha \partial g} + 1) \tag{131}$$

In what follows, we derive conditions for the concavity of the service provider's objective function which is a sufficient condition for the existence of a unique Stackelberg equilibrium.

Substituting (129) into (131),

$$\frac{\partial \pi_{sp}^{sp}}{\partial g} = \beta - \gamma(\frac{w(g)}{\alpha} + g) - \gamma g \frac{\alpha^2 I''(R(g))}{-\gamma \Delta^2 + 2\alpha^2 I''(R(g))} \tag{132}$$

Differentiating Equation (132) with respect to g we obtain,

$$\frac{\partial^2 \pi_{sp}^{sp}}{\partial g^2} = -2\gamma(\frac{\partial w(g)}{\alpha \partial g} + 1) - \gamma g \frac{\partial}{\partial g} \frac{\alpha^2 I''(R(g))}{-\gamma \Delta^2 + 2\alpha^2 I''(R(g))} \tag{133}$$

$$\frac{\partial}{\partial g} \frac{\alpha^2 I''(R(g))}{-\gamma \Delta^2 + 2\alpha^2 I''(R(g))} = \frac{-\gamma \Delta^2 \alpha^2 I'''(R(g)) \frac{\partial R(g)}{\partial g}}{\left(-\gamma \Delta^2 + 2\alpha^2 I''(R(g))\right)^2} \tag{134}$$

Substituting (126) into (134),

$$\frac{\partial}{\partial g} \frac{\alpha^2 I''(R(g))}{-\gamma \Delta^2 + 2\alpha^2 I''(R(g))} = \frac{\alpha^3 \gamma^2 \Delta^3 I'''(R(g))}{(-\gamma \Delta^2 + 2\alpha^2 I''(R(g)))^3} \tag{135}$$

The first term of the RHS of (133) is negative since $\frac{\partial w(g)}{\alpha \partial g} + 1 = \frac{\partial p_{sp}(g)}{\partial g}$ which we already

showed as positive. From (135) the second term of (133) is negative if $I'''(R) \geq 0$ as

$\frac{\gamma\Delta^2}{2\alpha^2} < I''(R)$. Hence, the service providers objective function is concave and there exists a

unique Stackelberg equilibrium if $I'''(R) \geq 0$ and $\frac{\gamma\Delta^2}{2\alpha^2} < I''(R)$. In this case, the optimal

w_{sp}, g_{sp} and R_{sp} satisfied at the equilibrium are provided by equating the first derivatives

(118), (119) and (132) to zero.

6.2 Comparison of the Remanufacturabilities and Prices in SPDSC and MDSC

PROPOSITION 4. *When product life is constant, MDSC has a lower remanufacturability and higher price per unit service to the customers compared to SPDSC.*

Proposition 4 suggests that SPDSC should be encouraged if a greater level of remanufacturing and lower price per unit service to the customers are desired. An interpretation for the proposition is as follows. The service provider being the follower in MDSC has no incentive to change her reaction function from the case when there is no remanufacturing. Hence, only the manufacturer who is the leader reduces his wholesale price from the case when there is no remanufacturing. In SPDSC, the follower (manufacturer) considers the cost savings from remanufacturing and chooses his reaction function so as to realize a greater demand for products and cost savings from remanufacturing. The service provider being the leader chooses his margin considering the manufacturer's reaction. In this sense, unlike in MDSC, both the supply chain members consider the cost savings due to remanufacturing in their profit maximization in SPDSC

Proof: We will first show that the optimal remanufacturability in MDSC R_m is less than the optimal remanufacturability in SPDSC R_{sp}. We will then $R_m < R_{sp}$ to show that $p_m > p_{sp}$. Let us first consider the case of SPDSC. Equating the first derivative (132) to zero,

$$\frac{\partial \pi_{sp}^{sp}}{\partial g} = \beta - \gamma(\frac{w(g)}{\alpha} + g) - \gamma g \frac{\alpha^2 I''(R(g))}{-\gamma\Delta^2 + 2\alpha^2 I''(R(g))} = 0 \tag{136}$$

Setting the first order condition (118) to zero we obtain,

$$w_{sp} = \frac{\alpha\beta + C_m\gamma - R_{sp}\gamma\Delta}{2\gamma} - \frac{\alpha g_{sp}}{2} \tag{137}$$

Substituting (137) into (136),

$$(\frac{\alpha\beta - C_m\gamma + R_{sp}\gamma\Delta}{2\alpha}) - \gamma g(\frac{-\gamma\Delta^2 + 2\alpha^2 I''(R_{sp}) + 2\alpha^2 I''(R_{sp})}{2(-\gamma\Delta^2 + 2\alpha^2 I''(R_{sp}))}) = 0 \tag{138}$$

$$(\frac{\alpha\beta - C_m\gamma + R_{sp}\gamma\Delta}{2\alpha}) = \gamma g(\frac{-\gamma\Delta^2 + 4\alpha^2 I''(R_{sp})}{2(-\gamma\Delta^2 + 2\alpha^2 I''(R_{sp}^*))}) \tag{139}$$

$$g^* = \frac{(\alpha\beta - C_m\gamma + R_{sp}\gamma\Delta)(2\alpha^2 I''(R_{sp}) - \gamma\Delta^2)}{\alpha\gamma(4\alpha^2 I''(R_{sp}^*) - \gamma\Delta^2)} \tag{140}$$

Equating the first derivative (119) to zero, we have

$$\frac{\partial \pi_{sp}^m}{\partial R} = \frac{(\beta - \gamma(g_{sp} + \frac{w_{sp}}{\alpha}))\Delta}{\alpha} - I'(R_{sp}) = 0 \tag{141}$$

Substituting (137) into (141),

$$\frac{\Delta\left(\alpha\beta - C_m\gamma + R_{sp}\gamma\Delta\right)}{2\alpha^2} - \frac{\gamma\Delta g_{sp}}{2\alpha} = I'(R_{sp}) \tag{142}$$

Substituting (140) into (142) and solving for $I'(R_{sp})$, we obtain

$$\frac{\Delta}{2}\left(\frac{\left(\alpha\beta - C_m\gamma + R_{sp}\gamma\Delta\right)}{\alpha^2} - \frac{(\alpha\beta - C_m\gamma + R_{sp}\gamma\Delta)(2\alpha^2 I''(R_{sp}) - \gamma\Delta^2)}{\alpha^2(4\alpha^2 I''(R_{sp}) - \gamma\Delta^2)}\right) = I'(R_{sp}) \tag{143}$$

$$I'(R_{sp}) = \frac{\Delta(\alpha\beta - C_m\gamma + R_{sp}\gamma\Delta)I''(R_{sp})}{(-\gamma\Delta^2 + 4\alpha^2 I''(R_{sp}))} \tag{144}$$

Now, let us consider manufacturer's profit in MDSC π_m if the remanufacturability R is set to R_{sp} Then equating the first derivative (7) to zero and solving for w, the optimal wholesale price $w_{|R=R_{sp}^*}$ satisfies,

$$w_{|R=R_{sp}^*} = \frac{\alpha\beta + C_m\gamma - R_{sp}\gamma\Delta}{2\gamma} \tag{145}$$

The rate of change of profit with increase in R when at $R = R_{sp}$ and $w_{|R=R_{sp}}$ is obtained by substituting the value of $w_{|R=R_{sp}^*}$ from into the first derivative (8),

$$\frac{\partial\pi_m^m}{\partial R_{R_{sp}^*,w|R=R_{sp}^*}} = \frac{\Delta\left(\alpha\beta - C_m\gamma + R_{sp}\gamma\Delta\right)}{4\alpha^2} - I'(R_{sp}) \tag{146}$$

Substituting the value of $I'(R_{sp})$ from (144) into (146)

$$\frac{\partial \pi_m^m}{\partial R_{R_m^*, w|R=R_{sp}^*}} = \frac{\gamma\Delta^3 (\alpha\beta - C_m\gamma + R_{sp}\gamma\Delta)}{4\alpha^2 (\gamma\Delta^2 - 4\alpha^2 I''(R_{sp}))} \tag{147}$$

$(\alpha\beta - C_m\gamma + R_{sp}\gamma\Delta) > 0$ if the optimal demand in SPDSC is greater than zero. Since

$\gamma\Delta^2 - 4\alpha^2 I''(R_{sp}) < 0$ by assumption about concavity, from (147), $\frac{\partial \pi_m}{\partial R} < 0$ at $R = R_{sp}$ and

$w_{|R=R_m^*}$. Therefore, if we set $R = R_{sp}$ in MDSC, the manufacturer can increase his profits by

decreasing R. However, as π_m is a convex function of R and w, $R_m < R_{sp}$.

We will now utilize $R_m < R_{sp}$ to prove that the equilibrium price in MDSC p is greater than

or equal to the optimal price in SPDSC p_{sp}. The price in SPDSC $p_{sp} = g + \dfrac{w_{sp}}{\alpha}$.

Substituting for the w_{sp} from (137) and g from (140).

$$p_{sp} = \frac{\beta\gamma\Delta^2 + \alpha(-3\alpha\beta - C_m\gamma + R_{sp}\gamma\Delta)I''(R_{sp})}{\gamma(\gamma\Delta^2 - 4\alpha^2 I''(R_{sp}))} \tag{148}$$

From (10) and the service provider's best response function (3)

$$p_m = \frac{3\alpha\beta + C_m\gamma - R_m\gamma\Delta}{4\alpha\gamma} \tag{149}$$

From (148) and (149),

$$p_m - p_{sp} = \frac{(\Delta^2 (\alpha\beta - C_m\gamma + R_m\gamma\Delta) + 4\Delta\alpha^2 (R_{sp} - R_m)\alpha^2 I''(R_{sp}))}{4\alpha(-\gamma\Delta^2 + 4\alpha^2 I''(R_{sp}))} \tag{150}$$

The first term of the numerator of (150), $(\alpha\beta - C_m\gamma + R_m\gamma\Delta)$ is positive if the optimal

demand in MDSC is positive. The second term is positive since $R_m < R_{sp}$. Hence, $p_{sp} < p_m$

. The above result shows that under some general investment functions *I(R)* and with linear

demand, SPDSC leads to a higher remanufacturability, higher number of products (lower

price) and greater quantities of remanufactured products as compared to MDSC.   We will

now compare the remanufacturabilities and prices in SPDSC and CCSC.

6.3 Comparison between SPDSC and CCSC.

In this section, we show that the optimal remanufacturability in SPDSC R_{sp} is less than the

optimal remanufacturability in CCSC R_c . Suppose, instead of the optimal R_c, we set the

remanufacturability at $R = R_{sp}$ in CCSC and then find the best price *p* given that $R = R_{sp}$ by

setting the first derivative given by (35) to zero,

$$p_{|R=R_{sp}^*} = \frac{\alpha\beta + \gamma(C_m - R_{sp}\Delta)}{2\gamma\alpha} \tag{151}$$

Then, the rate of change of profit π_c with increase in R at $R = R_{sp}$ and $p_{|R=R_{sp}^*}$ is given by

equating the first derivative (36) to zero and substituting for $p_{|R=R_{sp}^*}$ from (151)

$$\frac{\partial \pi_c}{\partial R_{R=R_{cp}^*, P|R=R_{sp}^*}} = \frac{\Delta\left(\alpha\beta - C_m\gamma + R_{sp}\gamma\Delta\right)}{2\alpha^2} - I'(R_{sp}) \tag{152}$$

Substituting the value of $I'(R_{sp})$ from (142) into (152),

$$\frac{\partial \pi_c}{\partial R_{R=R_{sp}^*,\, p|R=R_{sp}^*}} = \frac{\gamma \Delta g}{2\alpha} \tag{153}$$

Therefore, from (153) if $R = R_{sp}$ in CCSC, we can increase the total profit π_c by increasing

R. However as π_c is a concave function of R and p, $R_c > R_{sp}$. We will now show that the

optimal price in SPDSC p_{sp} is greater than the optimal price in CCSC p_c. Considering

(137) and (35),

$$p_{sp} = \frac{w_{sp}}{\alpha} + g \quad = \frac{\alpha\beta + C_m\gamma - R_{sp}\gamma\Delta}{2\alpha\gamma} + \frac{g}{2} \tag{154}$$

$$p_c = \frac{\alpha\beta + C_m\gamma - R_c\gamma\Delta}{2\alpha\gamma} \tag{155}$$

$$p_c - p_{sp} = \frac{\Delta}{2\alpha}(R_{sp} - R_c) - \frac{g}{2} \tag{156}$$

From (156) $p_c - p_{sp} < 0$ since $R_{sp} - R_c < 0$.

The above results show that the lack of coordination (Double marginalization) leads to a

higher price for the manufacturer and lower level of remanufacturing when service provider

is the Stackelberg leader. The optimal remanufacturability depends optimal number of

products and the cost saving due to remanufacturing Δ. One interpretation of this proposition

is that the double marginalization in this decentralized SC leads to decreased demand and

thus a lesser incentive to invest in remanufacturing.

We will show that the total SC profit of SPDSC is higher than that of MDSC.

Proof: We will provide the proof as Lemma 1 and Lemma 2.

LEMMA 1: The maximum profit that can be achieved given a price p and optimizing over R in CCSC is a concave function of the price p.

Proof: Let $f(x,y)$ be a concave function of x and y defined over C a convex non-empty set. Let the function $g(x)$ be the maximum of $f(x,y)$ given x and maximizing y over C.

$$g(x) = \sup_{y \in c} f(x, y) \tag{157}$$

Let $\varepsilon > 0$. Then there are y_1 and $y_2 \in C$ such that $f(x_i, y_i) \geq g(x_i) - \varepsilon$ for $i = 1, 2$.

$$
\begin{aligned}
g(\theta x_1 + (1-\theta)x_2) &= \underset{y \in c}{su} \, f(\theta x_1 + (1-\theta)x_2, y) \\
&\geq f(\theta x_1 + (1-\theta)x_2, \theta y_1 + (1-\theta)y_2) \\
&\geq \theta f(x_1, y_1) + (1-\theta)f(x_2, y_2) \quad by \, concavity \\
&\geq \theta g(x_1) + (1-\theta)g(x_2) - \varepsilon
\end{aligned}
\tag{158}
$$

Since the above holds for any value of $\varepsilon > 0$, we have that

$$g(\theta x_1 + (1-\theta)x_2) \geq \theta g(x_1) + (1-\theta)g(x_2) \tag{159}$$

Hence, $g(x)$ is a concave function. Applying the above lemma, the maximum of total SC profit given a price p and optimizing over R is a concave function of p. A similar proof for the case of minimization and convex function is given in Boyd (2004).

LEMMA 2: The total SC profit in MDSC and SPDSCs is equal to the maximum total SC profits that can be obtained given the optimal prices in the respective SC's.

Proof: Suppose we are maximizing the total SC profits with respect to R given a fixed price p_c. Then the first order conditions with respect to R is as follows

Coordinated SC (total SC profit maximization):

$$\frac{(\beta - \gamma p_c)\Delta}{\alpha} - I'(R_c) = 0 \tag{160}$$

First order conditions in MDSC and SPDSC obtained by equating (8) and (119) to zero can be simplified as shown in (161) and (162)

$$\frac{(\beta - \gamma p_m)\Delta}{\alpha} - I'(R_m) = 0 \tag{161}$$

$$\frac{(\beta - \gamma p_{sp})\Delta}{\alpha} - I'(R_{sp}) = 0 \tag{162}$$

Comparing (160) and (161), we observe that R_m maximizes the total SC profits given p_m.

Similarly comparing (160) and (162), R_{sp} maximizes the total SC profits given the p_{sp}.

From Lemma 1, the total SC profits given a price p and optimizing over R (i.e., the objective of CCSC) is a concave function of price p. The maximum of this function is achieved at price p_c. Hence, the total SC profit decreases with p when $p \geq p_c$. Given $p_c^* < p_{sp}^* < p_m^*$ and considering Lemma 2, the total SC profit in SPDSC is greater than the total SC profit in MDSC. Hence, $\pi_c > \pi_{sp}^{total} > \pi_m^{total}$. Considering that SPDSC also leads to a lower price to the consumers, the total surplus of SPDSC is greater than the total surplus of MDSC.

6.4 Numerical Example with Product Life as a Parameter

The parameters of this numerical example are the same as Section 3.52. The specific

parameter values are $k = 1\ 000\ 000$, $C_m = 23737$, $\beta = 8\ 000\ 000$ and $\gamma = 2\ 500\ 000$. We first

varied Δ between 2000 and 7000 with a fixed α of 12 113. Figure 27 and

Figure 27 and Figure 28 show the variation of R and p with the cost savings per

remanufactured product Δ. We observe that the R and p in MDSC and SPDSC are very quite

similar. Figure 28 Variation of p with Δ under MDSC, CCSC and SPDSC

Figure 29shows the variation of ζ with Δ in various SC. We can observe that the ζ for

SPDSC is 33%, same as for the forward supply chain shown in Appendix B. When Δ is high

the combined effect of reduced price and increased remanufacturing are leading to higher
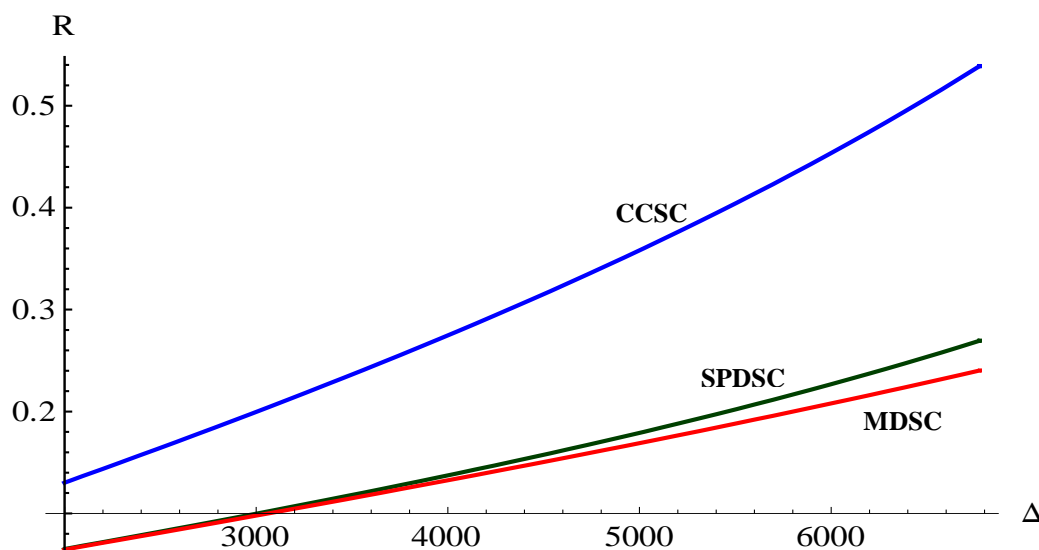
profits SPDSC compared to MDSC.

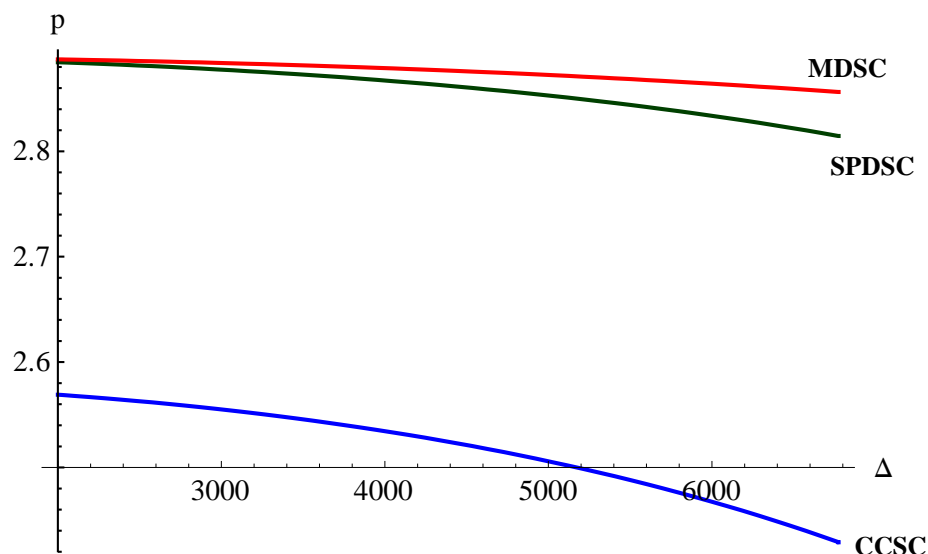**Figure 27 Variation of *R* with Δ under MDSC, CCSC and SPDSC**

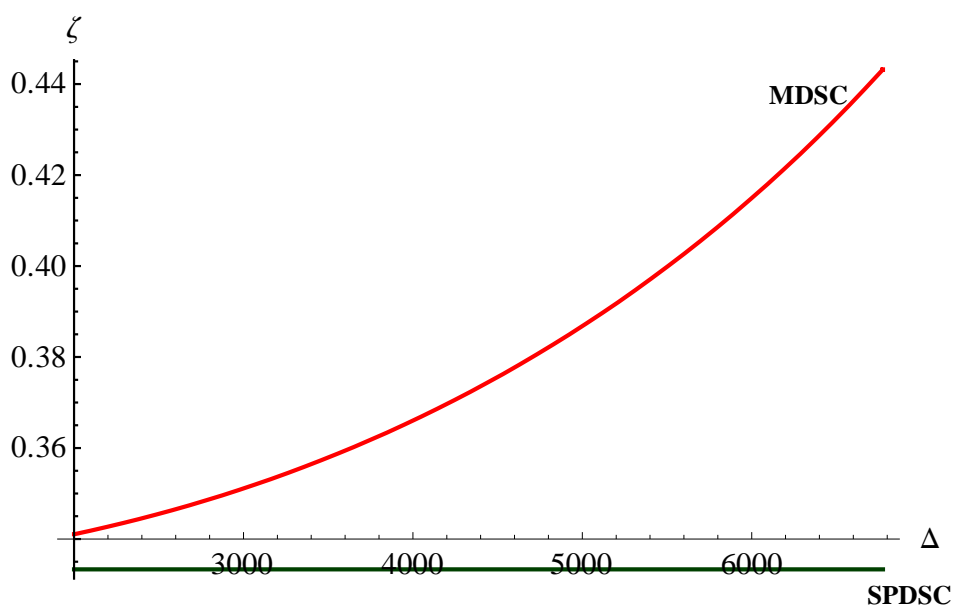**Figure 28 Variation of *p* with Δ under MDSC, CCSC and SPDSC**

Figure 29 Variation of the Percentage Gain in Profit by Coordinating MDSC and SPDSC with Δ.

6.5 Numerical Example with Product Life as a Variable

Being the follower, the manufacturer accepts the service provider's margin and then determines the wholesale price *p*, remanufacturability *R* and the product life *α* that maximize his profits. The service provider considers the manufacturer's reaction *w* (*g*) and chooses the

margin that optimizes her margin considering w (g). The manufacturer's profit maximization

problem is formulated by (163).

$$\underset{w, R, \alpha}{Max} \pi_m = \frac{(\beta - (g + \frac{w}{\alpha})\gamma)}{\alpha}(w - C_m(\alpha) + R\Delta) - I(R) \tag{163}$$

Equation (164) and (164) are sufficient conditions for the quasiconcavity of the

manufacturers profit function π_m and the existence of the Stackelberg equilibrium.

$$\Delta^2(\beta - g\gamma) - \gamma\Delta^2 C_m'(\alpha\ ) - \alpha\ C_m''(\alpha^*)(4\alpha^{*2}I''(R\ ) - \gamma\Delta^2)) < 0 \tag{165}$$

$$2\alpha^2 I''(R\ ) > \gamma\Delta^2 \tag{166}$$

In case of SPDSC with a variable α analytic conditions for quasiconcavity of the

manufacturer's objective was hard to interpret. Hence, we will observe how SPDSC solution

differs from other supply chains in a numerical example. The parameters of this example are

the same as in Section 4.5. The specific values utilized are as follows: k = 1000 000, β=800

0000, γ = 2 500 000, C_{m0} =12 000, s = .00008. The cost savings per remanufactured product

was varied between 2000 and 6500.

In this numerical example, we obtained SPDSC equilibrium by solving the first order

conditions. Given the specific parameter values, we obtained only one critical point where

the first order conditions were met where α>0 and1 >R>0. The uniqueness of the equilibrium

was confirmed by testing whether the service providers profit is concave at the critical point.

Figures 30, 31 and 32 show the variation of *R*, *p,* and α in various supply chain. We can

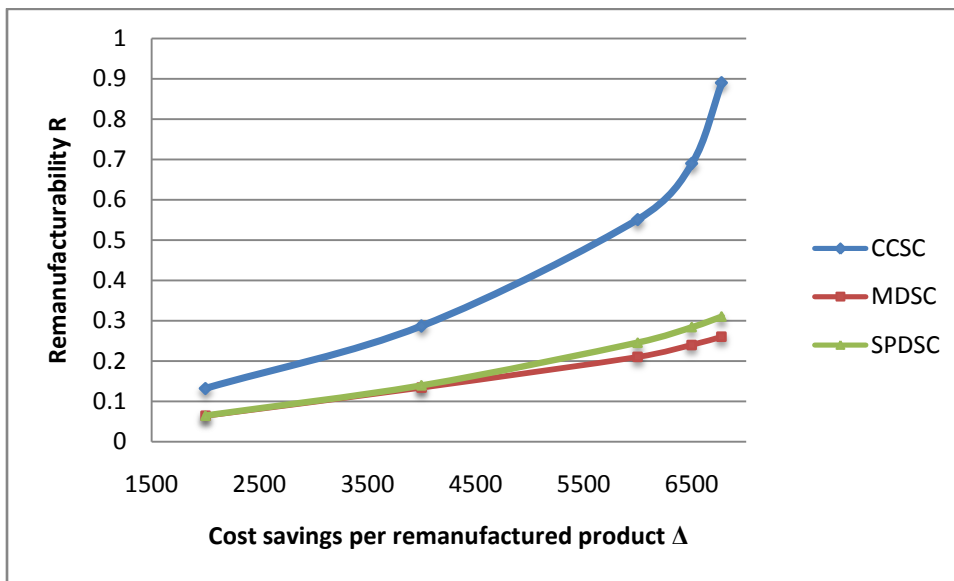observe that SPDSC solution is quite similar to MDSC solution.

Figure 30 Variation of the R with Δ in MDSC, CCSC, and SPDSC when α is Variable
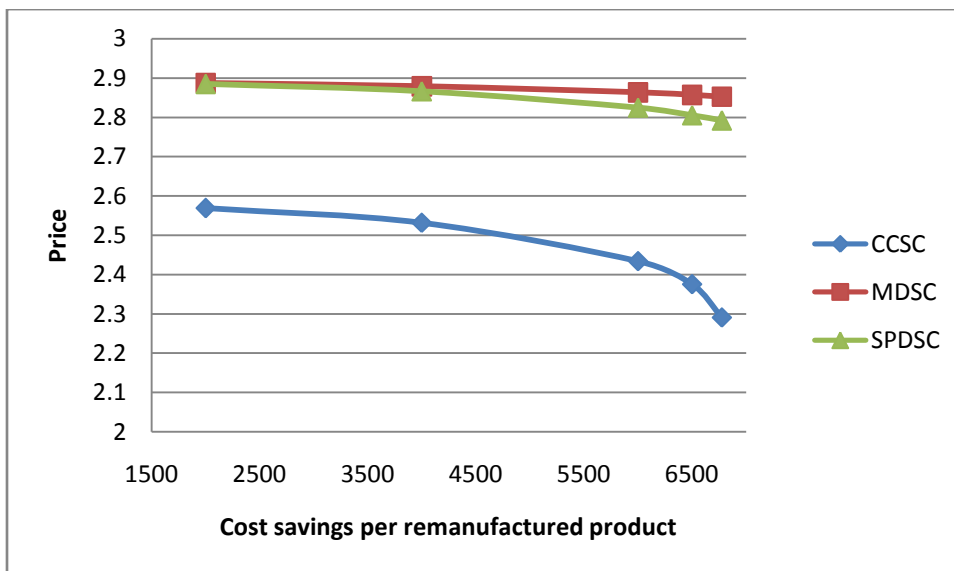

Figure 31 Variation of the Price p with Δ in MDSC, CCSC, and SPDSC when α is Variable
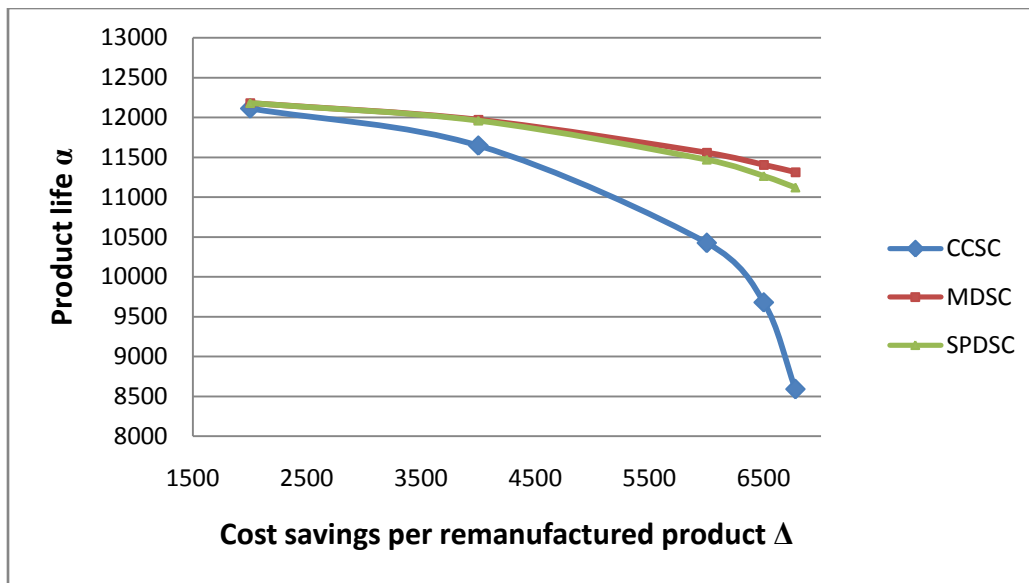
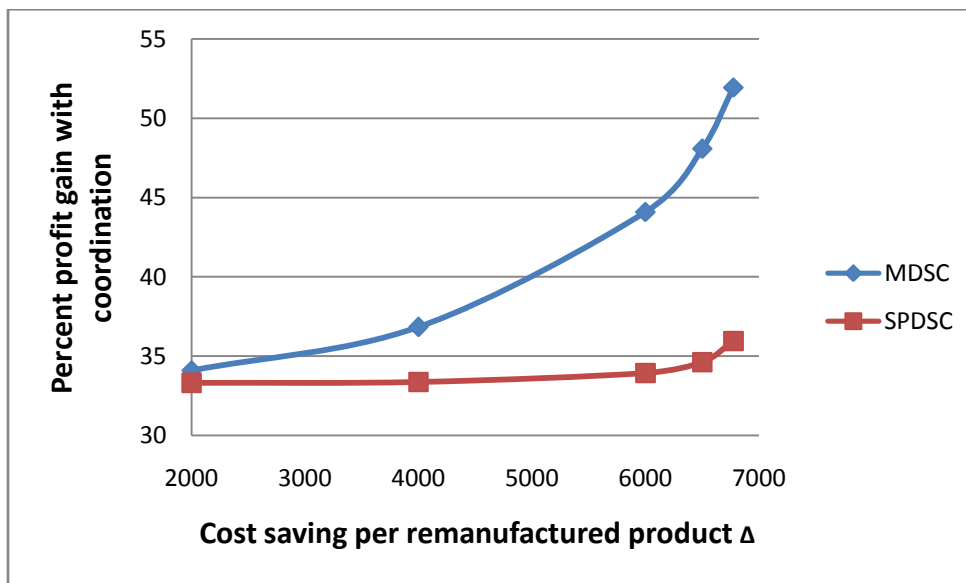**Figure 32 Variation of the α with Δ in MDSC, CCSC, and SPDSC when α is Variable**

Figure 33 Variation of the Percent Profit Gain with Coordination ζ with Δ

7. Concluding Remarks

The purpose of this study was to model and analyze the economic relationships among the level of remanufacturing, product life, and economic consequences under the framework of a manufacturer/remanufacturer and a service provider who utilizes the manufacturer's product to provide service to her customers. In addition, the study had the following research objectives.

1) To derive the relationships between the remanufacturability and product life in the context of a manufacturer-service provider supply chain.

2) To investigate how supply chain coordination between the manufacturer and service provider impacts the remanufacturability, product life and prices.

3) To analyze the impact of an environmental legislation that penalizes disposal of products with respect to the level of remanufacturing and product life.

To achieve these objectives,

In Chapter 3, we formulated MDSC when product life is a parameter as a Stackelberg game, performed comparative statics of the equilibrium and compared MDSC equilibrium with the benchmark CCSC solution. Assuming linear demand and a general investment function, we found that the lack of coordination in MDSC is resulting in a higher price per unit service and lower investment in remanufacturability compared to CCSC. Our results are a generalization of similar results obtained by other articles to the case of service supply chain and a more general class of investment functions.

In Chapter 4, we formulated MDSC with a variable product life as a Stackelberg game, performed comparative statics analysis of the equilibrium and compared MDSC equilibrium with the benchmark CCSC solution. Our results showed that several conditions resulting in a higher remanufacturability such as increased cost saving per remanufactured product and reduced slope of the demand function also lead to lower product life. The managerial implication of this result is that manufacturers/remanufacturers should consider a high remanufacturability low product life strategy as a business option under such conditions. While CCSC solution has a higher remanufacturability compared to MDSC equilibrium, it has a lower product life. Hence, the decentralized MDSC might be preferred from the environmental stand point if a longer product life is desired.

In Chapter 5, we analyzed how an environmental fee imposed upon each disposed product influences MDSC equilibrium when product life is constant. We found that under some conditions when the marginal demand is sufficiently high, the environmental fee can counter intuitively result in reduced investment in remanufacturability and a higher price. Given variable product life, we observed from a numerical example that increase in the environmental fee can simultaneously result in increased product life as well as remanufacturability in the MDSC.

In Chapter 6, we formulated SPDSC as a benchmark for the scenario where the service provider is the dominant supply chain member. With product life as a parameter, we found that SPDSC is superior to MDSC with respect to total supply chain profits, remanufacturability and the price per unit service. With product life as a variable, we

observed that SPDSC has a higher remanufacturability and a lower product life compared to MDSC.

Thus far, we have summarized our investigational achievements. We will now discuss items for future study.

The gaming process assumed in SPDSC is formulated in terms of margins and is less intuitive than MDSC. One of the concerns is whether the manufacturer's technology choices are influenced by the service provider pricing decisions. In case of a traditional supply chain, anecdotal evidence suggests that powerful retailers such as Walmart can bring about a greater demand for products and lead to investments by manufacturers in production cost saving technology (e.g, reusable packaging). Validating the above results with empirical research on the gaming process in SPDSC is an interesting area for further study.

We assumed a single period model in which the decisions are all made at the start of the period. The length of the period was defined as the operation period of the investment in remanufacturability. In a dynamic model where the prices and production quantities can be vary across periods, the investment in remanufacturability and product life can both influence the future cash flows and can be impacted by the discount factor. Investigating, whether the relationship between product life and remanufacturability identified in this research are valid under a dynamic setting is an interesting area for further research.

We modeled fixed cost investment in remanufacturability and product life as a function of manufacturing cost. In reality, the remanufacturability may also be increased in some cases by utilizing better quality long life components as discussed by [34]. Similarly product life

may be improved via a more robust design. Investigating the impact of these alternate models is another important area for further study.

We assumed a linear demand to characterize the key supply chain phenomena in a tractable manner. Given the linear demand, the sensitivity analysis of the equilibrium was presented in terms of the parameters of the linear demand. Extending the model to other demand functions such as the constant elasticity demand and analyzing how the demand elasticity impacts the product life and remanufacturability is an important avenue for further study.

Finally, we assumed that the manufacturer and service provider have a monopoly in the product and services offered. Investigating the impact of competition in alternative supply chain configurations with multiple manufacturers and service provider's [11, 29, 38] is an interesting area for further research.

These issues under the discussion will provide interesting and relevant future studies. We think that the models and the analysis presented in this chapter will serve as a strong basis for such extensions.

Appendix A: Summary of Notation

π_m^m is the manufacturer's profit in MDSC (dollars).

π_{sp}^m is the service provider's profit in MDSC(dollars).

π_m^{sp} is the manufacturer's profit in the service provider driven supply chain (dollars).

π_{sp}^{sp} is the service provider's profit in the service provider driven supply chain(dollars).

w is the wholesale price paid by the service provider to the manufacturer for each product (dollars/product)

α the product life is the number of units of service provided by a product from its manufacturer/ remanufacture to when it is collected as a used product for possible remanufacturing (units of service/ product).

C_m is the manufacturing cost per product (dollars/product).

C_r is the remanufacturing cost per product (dollars/product).

Δ is the cost saving per remanufactured product (dollars/product). i.e. $\Delta = C_m - C_R$

R the remanufacturability is the fraction of used products that are remanufactured.

$I(R)$ is the investment required to achieve a level of remanufacturability of R (dollars).

p is the price per unit service charged by the service provider to the customers (dollars/ unit service)

β is the maximum demand parameter of the linear demand function (units service). β is the demand when p is zero.

γ is the marginal demand parameter of the linear demand function ((unit service)^2/dollar). γ is the decrease in demand when the price increases by a dollar.

Appendix B: The Manufacturer Driven Forward Supply Chain without Remanufacturing

This supply chain is MDSC where remanufacturing is not possible. Several articles in the literature have addressed the forward supply chain [13]. The calculation of the equilibrium values for the case of the linear demand is straightforward. We provide expressions for the equilibrium values that are useful as benchmarks for comparing the results with remanufacturing.

The equilibrium wholesale price in the w_{mf} is

$$w_{mf} = \frac{\alpha\beta + C_m\gamma}{2\gamma} \qquad \text{(B.1)}$$

The equilibrium price in the p_{mf} is

$$p_{mf} = \frac{3\alpha\beta + C_m\gamma}{4\alpha\gamma} \qquad \text{(B.2)}$$

The equilibrium demand is

$$\beta - \gamma p_{mf} = \frac{\alpha\beta + C_m\gamma}{4\alpha} \qquad \text{(B.3)}$$

$$\pi^*_{mf} = \frac{(\alpha\beta - C_m\gamma)^2}{8\alpha^2\gamma} \qquad \text{(B.4)}$$

$$\qquad \text{(B.5)}$$

$$\pi^*_{spf} = \frac{(\alpha\beta - C_m\gamma)^2}{16\alpha^2\gamma}$$

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